

Problem 1: The Mandelbrot sequence

We study here the sequence given by:

$$z_{n+1} = z_n^2 + c$$

with the initial condition z_0 is equal to zero, and “ c ” is a given complex parameter. The sequence is complex (z_n are complex numbers) when the constant c has an imaginary part $\text{Im}(c) > 0$ or $\text{Im}(c) < 0$.

The Mandelbrot sequence plays an important role in complex dynamics, a field investigated by the mathematician Pierre Fatou and Gaston Julia at the beginning of the 20th century.

1. Write a program by using a “do loop” to compute the convergence point of the sequence for $c=(0.1,0.2)$, i.e. the real part of c is 0.1, and the imaginary part of c is 0.2 We assume in this problem that the sequence reaches the converge for $n=200$. To test your program, you should find that the convergence point is (0.052786,0.0223606). As a hint, the idea here is to decompose z_n in its real and imaginary part (a_n, b_n). The real and imaginary part of the coefficient z_{n+1} can then be computed separately (a_{n+1}, b_{n+1}).
2. Repeat the same operation many times for different values of the constant c . For this, use two nested do loops to repeat the calculation of the sequence for constant c values in a grid spanning $[-2,1]$ for the real part of c , and $[-1,1]$ for the imaginary part of c .

```
n1=500
```

```
do j1=1,n1
```

```
do j2=1,n1
```

```
Cre=-2.+3.*dble(j1-1)/dble(n1-1)
```

```
Cim=-1.+2.*dble(j2-1)/dble(n1-1)
```

3. Find a strategy to detect when the sequence is converging or not. As a criteria, we can assume that the sequence does not converge when the absolute value of z_n is larger than 10.
4. Write into a file fort.100 the values of the constant c for which the sequence is converging. The file fort.100 will have two columns, the first column will contain the real part of the constant c for which the sequence is converging, and the second column the imaginary part.

5. Plot the result with xmgrace. Note: you might get an error message saying that the number of lines is too large, simply discard this message. Double click on the obtained line, click on the menu "Line properties, Type" and replace "Straight" with "None". Then choose the menu "Symbol properties Type", and replace "None" with "circle", and change the size cursor below to "Size 8". Export this graphic to an image file (jpg) that you can include in your report.
6. Repeat 5 with a grid of constant c spanning the range $[-1.5, -1.2]$ for the real part and $[-0.1, 0.1]$ for the imaginary part.