Problem 1: The expanding civilization

We study here the sequence given by:

$$x_{n+1} = a * x_n * (1 - x_n)$$

with the initial condition x_0 chosen in the interval [0,1], and "**a**" is a given parameter between 0 and 4.

This sequence is a simple model to describe how a population (or civilization) evolves in a closed system with a finite amount of resources. The parameter "**a**" describes the rate of development and expansion of the civilization. The population is described by a simple variable x, which quantifies the total population (when x=0 the population is naught, when x=1 the population has reached its maximum sustainable size).

The civilization grows with the combined rate of reproduction (proportional factor **x** in the sequence) and starvation (term **1-x**). Indeed, since the resources are limited the expansion is also proportional to "**(1-x)**" and if the civilization reaches the maximum population **x**=1 it will have exhausted all its resources and then collapses (**x**=0).

1. Write a program by using a "do loop" to compute the convergence point of the sequence for a=1, and a random number between 0 and 1 for x_0. We assume in this problem that the sequence reaches the converge point before n=100, so the convergence point x* can be approximated by the n=100 element of the sequence (x_{100}).
2. Write into a file the value of "a" and the obtained convergence point x*
3. Add a do loop to repeat the same operation for 50 different random initial conditions. All obtained pairs of data points (a, x*) should be written into the Fortran file.
4. Add one more do loop to repeat the same procedure (1-3) for N=2'000 different rate parameters "**a**" spanning the interval [1.5,4]. This means that "a" needs to be equal to a=1.5 for the first cycle of your additional "do loop", and be equal to "a=4" for the last cycle of this additional "do loop".
5. Plot the result with xmgrace. Note: you might get an error message saying that the number of lines is too large, simply discard this message. Double click on the obtained line, click on the menu "Line properties, Type" and replace "Straight" with "None". Then choose the menu "Symbol properties Type", and replace "None" with "Star", and change the size cursor below to "Size 8". You should have obtained a graphic of the fix points x* of the sequence x_n obtained for different growing rate **a**. These plots can be exported to an image file (jpg) that can be included in your report. Repeat the same procedure (4-5) with a range of **a** in [3,4] and then [3.5,3.8].

Section Problem 1:

1. Explain briefly what your approach is to generate a sequence, how it is realized within the general structure of a computer code. Feel free to write a few lines of code to illustrate your point.
2. Discuss now which are the different "do-loops" involved in the code, and what they achieve, what is their purpose?
3. Using the code you have written, obtain and include in your report, a graphic which shows on the horizontal axis the growing rate "**a**" and on the vertical axis the obtained fix points "**x***" for the set of initial random x_0 values. Show one graphic with **"a"** ranging from 1.5 to 4, and then also with "**a**" ranging from 3 to 4, and 3.5 to 3.8. Show the three plots in your report.
4. For **a**=1.5 there is only one fix point. For which values of a do you obtain more than one fix point?
5. Discuss the observed trend as "a" is increasing, and the consequence of the accumulation of fix points for the stability of the growing civilization.
6. The plots that you obtain in (3) have similar structures, this is called self-similarity. Do a quick literature search to discuss this concept and its connection with the sequence discussed in this problem.