Exercise 1: Fibonacci

In this problem, we study the Fibonacci series, written as: $x_n = x_{n-1} + x_{n-2}$, with initial conditions : $x_0=1$, $x_1=a$. The Fibonacci series is obtained for a=1, and gives:

1,1,2,3,5,8,13,21,34,55,89,144,233.....

I. Write a new program which displays the 100 first elements (n=100) of the Fibonacci series for a=1

II. generalize your program, such that the user can enter the variable "a" and the maximum number of computed elements n_{max} in the keyboard

III. Display the ratio $r_n = x_{n+1}/x_n$ for several increasing values of n. What do you observe?

IV. Repeat (I) above with a different value of "a" (a=5), how does r_n compare to what was obtained in (I)?

V. How does r_n compare to $g=(1+\sqrt{5})/2$? g is the so-called golden number. Show the deviation between r_n and the golden number as n increases.

VI. Try a few starting condition "a" and see if you can obtain a different convergence for r_n than the Golden ratio. Is the Golden ratio a "stable" fix point or "unstable" fix point of the sequence r_n ? (does the sequence converge to the Golden ratio for a broad range of initial condition "a" or a very specific subset of conditions "a"?).

Exercise 2: Golden number

The golden number is closely related to the Fibonacci series, as hinted already in Exercise 1 above.

I. For a=1, can you find the analytical solution for the ratio r_n? Hint: a recurrence sequence can be written for r_n: $r_n = 1 + 1/r_{n-1}$, with initial condition $r_1=a$, and n=2,3,4... (look in the lecture note for an example how to do that).

II. How many analytical solutions do you find for r_n above? If you find several ones, which one does correspond to the limit r_n obtained in Exercise (1) above?

III. Fix Points are points of convergence of series which starts with different initial condition and converge to the same result. They play a very important role in the convergence of iterative processes. Try to change "a" significantly and see if you can locate a starting which does not converge to the Golden number.

IV. Try to start now with $a= (1-\sqrt{5})/2$. What does happen now? Do you find that this value of alpha is also a fix point? Use first a moderate $n_{max}=20$ and then a larger value such as $n_{max}=100$. What do you conclude?

V. In (IV) above, we explored the notion of "unstable convergence point", it is a point of convergence extremely sensitive to the initial condition, and any numerical imprecision will make the series deviate from this convergence point. This idea is at the basis of the notion of "Chaos analysis" in physics.