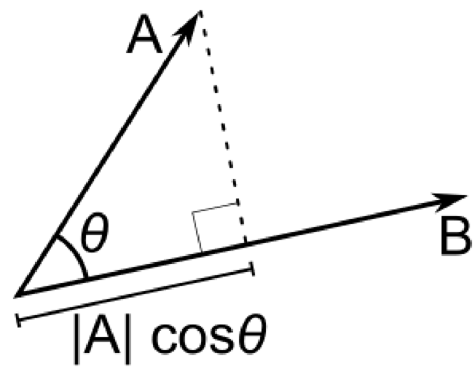
Kinematics

Math Essentials

Linear Algebra –Dot Product

The dot product of two vectors **A** = [A_1, A_2, ..., A_n] and **B** = [B_1, B_2, ..., B_n]

$$\mathbf{A}\cdot\mathbf{B}=\sum_{i=1}^{n}A_iB_i=A_1B_1+A_2B_2+\cdots+A_nB_n$$



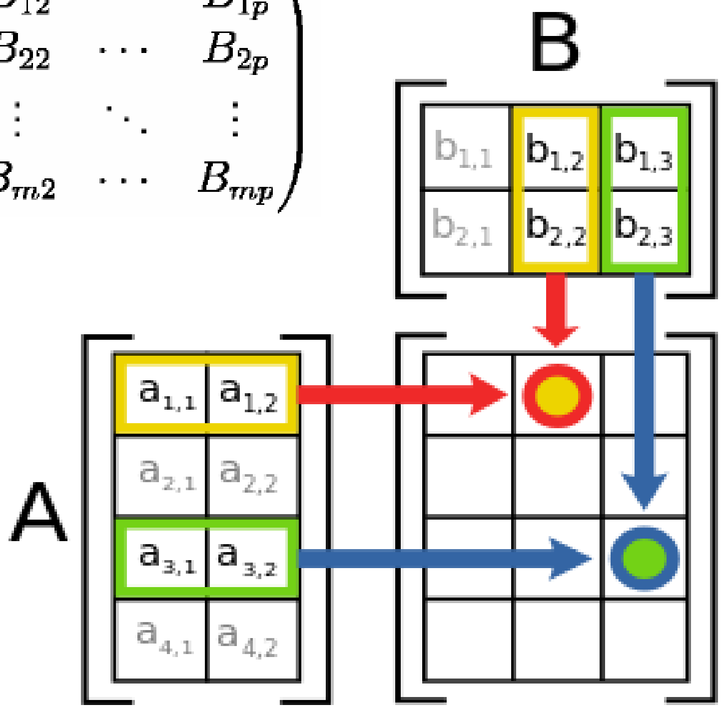
$$\mathbf{A}\cdot\mathbf{B}=\|\mathbf{A}\|\ \|\mathbf{B}\|\cos(\theta)$$

$\mathbf{A}\cdot\mathbf{B}=0$ if **A** and **B** are orthogonal

$\mathbf{A}\cdot\mathbf{B}=\|\mathbf{A}\|\ \|\mathbf{B}\|$ if they are codirectional

$$\mathbf{a}\cdot\mathbf{b}=\mathbf{a}^{\mathrm{T}}\mathbf{b}=\begin{pmatrix}a_1 & a_2 & \cdots & a_n\end{pmatrix}\begin{pmatrix}b_1\\ b_2\\ \vdots\\ b_n\end{pmatrix}=a_1b_1+a_2b_2+\cdots+a_nb_n$$

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1p} \\ B_{21} & B_{22} & \cdots & B_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ B_{m1} & B_{m2} & \cdots & B_{mp} \end{pmatrix}$$



$$(\mathbf{AB})_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj} = A_i \cdot B_j$$

Row vector and column vector

$$\mathbf{A} = \begin{pmatrix} a & b & c \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ax + by + cz$$

$$\mathbf{BA} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} a & b & c \end{pmatrix} = \begin{pmatrix} xa & xb & xc \\ ya & yb & yc \\ za & zb & zc \end{pmatrix}$$

Square matrix and column vector

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ px + qy + rz \\ ux + vy + wz \end{pmatrix}$$

Square matrices

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix} = \begin{pmatrix} a\alpha + b\lambda + c\rho & a\beta + b\mu + c\sigma & a\gamma + b\nu + c\tau \\ p\alpha + q\lambda + r\rho & p\beta + q\mu + r\sigma & p\gamma + q\nu + r\tau \\ u\alpha + v\lambda + w\rho & u\beta + v\mu + w\sigma & u\gamma + v\nu + w\tau \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix} \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} = \begin{pmatrix} \alpha a + \beta p + \gamma u & \alpha b + \beta q + \gamma v & \alpha c + \beta r + \gamma w \\ \lambda a + \mu p + \nu u & \lambda b + \mu q + \nu v & \lambda c + \mu r + \nu w \\ \rho a + \sigma p + \tau u & \rho b + \sigma q + \tau v & \rho c + \sigma r + \tau w \end{pmatrix}$$

$$\text{AB} \neq \text{BA}$$

$$\mathbf{A} = \begin{pmatrix} a & b & c \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{aligned}
\mathbf{ABC} &= \begin{pmatrix} a & b & c \end{pmatrix} \left[\begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right] = \left[\begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} \alpha & \beta & \gamma \\ \lambda & \mu & \nu \\ \rho & \sigma & \tau \end{pmatrix} \right] \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\
&= \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} \alpha x + \beta y + \gamma z \\ \lambda x + \mu y + \nu z \\ \rho x + \sigma y + \tau z \end{pmatrix} = \begin{pmatrix} a\alpha + b\lambda + c\rho & a\beta + b\mu + c\sigma & a\gamma + b\nu + c\tau \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\
&= a\alpha x + b\lambda x + c\rho x + a\beta y + b\mu y + c\sigma y + a\gamma z + b\nu z + c\tau z\,,
\end{aligned}$$

If $\mathbf{A}^T = \mathbf{C} = \mathbf{x}$

Then: $\mathbf{ABC} = \mathbf{x}^T\mathbf{A}\mathbf{x}$

Quadratic scaler function using matrix representation

Rectangular matrices

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ x & y & z \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \alpha & \rho \\ \beta & \sigma \\ \gamma & \tau \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} a & b & c \\ x & y & z \end{pmatrix} \begin{pmatrix} \alpha & \rho \\ \beta & \sigma \\ \gamma & \tau \end{pmatrix} = \begin{pmatrix} a\alpha + b\beta + c\gamma & a\rho + b\sigma + c\tau \\ x\alpha + y\beta + z\gamma & x\rho + y\sigma + z\tau \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} \alpha & \rho \\ \beta & \sigma \\ \gamma & \tau \end{pmatrix} \begin{pmatrix} a & b & c \\ x & y & z \end{pmatrix} = \begin{pmatrix} \alpha a + \rho x & \alpha b + \rho y & \alpha c + \rho z \\ \beta a + \sigma x & \beta b + \sigma y & \beta c + \sigma z \\ \gamma a + \tau x & \gamma b + \tau y & \gamma c + \tau z \end{pmatrix}$$

$$\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{A}\mathbf{B} + \mathbf{A}\mathbf{C}$$
$$(\mathbf{A}+\mathbf{B})\mathbf{C} = \mathbf{A}\mathbf{C} + \mathbf{B}\mathbf{C}$$

$$(\mathbf{A}\mathbf{B})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$$

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$(\mathbf{A}\mathbf{B})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

$$\det(\mathbf{A}\mathbf{B}) = \det(\mathbf{A})\det(\mathbf{B})$$