Topic 1
Engineering Economic Framework

Roadmap

1) Why Engineering Economics?
2) Analyzing Decision Using Engineering Economics
3) Interest Rate
4) Cash Flow Symbols and Cash Flow Diagram
5) Minimum Attractive Rate of Return (MARR)

1) Why Engineering Economics?

- People and firms make decisions - choosing one alternative over others
- Engineering economics involves formulation, estimation and evaluation of alternatives economically.
- Eng Econ = a set of tools in decision making
- Engineering decisions include: design choices, technology choices, procurement options, etc.
- After designing, engineers must justify projects, as everyone competes for a limited budget
- What are examples of individual decisions that can be addressed using engineering economics?

2) Analyzing Decision Using Engineering Economics

1. Define feasible alternatives
2. Estimate cash flows of each alternative
3. Evaluate each alternative using
   - Time value of money
   - Measures of economic worth
4. Select the best alternative

Note: In real-world decision making, the final choice may also depend on non-economic factors, such as regulation, politics, image, etc.

2.1) Defining feasible alternatives is problem specific.

- Each problem has at least one alternative of doing nothing, which may not be free.
- When the decision is
  1. Single project: accept or reject (do nothing)
  2. Multiple projects:
     1. *Mutually exclusive*: picking A means not picking B, C, etc.
     2. *Independent*: you can do several (good) projects at the same time, subject to budget limitation.

2.2) Estimating cash flows involve various parameters.

Common parameters in engineering economic analysis are:

- First cost or investment cost (purchase price + development + installation + …)
- Useful life
- Estimated annual income and expenses
- Salvage value (resale or trade-in value)
- Interest rate (rate of return)
- Inflation and tax effects, if applicable

2.3)&2.4) Evaluating and selecting the best alternative are core to engineering economics.

- Engineering economics is based on the concept of time value of money - a change in the amount of money over a given time period.
  - Money grows over time because money can generate money if invested.
  - This concept centers around an interest rate.
- With the time value of money, we compare alternatives using various economic measures, such as present value, annual value, rate of return, benefit/cost, etc.

3) Interest Rate

- Interest represents time value of money.
- Interest (I) = ending amount - beginning amount

$$\text{INTEREST RATE} = \frac{\text{INTEREST PER TIME UNIT}}{\text{ORIGINAL AMOUNT}}$$

- Ex Joe borrows $500,000 and must repay $530,000 exactly one year from now. Determine the interest and the interest rate.

  Interest =

  Interest rate =

3.1) Interest rate can be viewed from two perspectives.

1. Borrower's perspective = *interest rate paid*, for renting someone's money

   Ex You borrow $1,000 from your friend and will pay back $1,050 at the end of one year. What is an interest rate paid?

2. Investor's perspective = *rate of return* on your investment

   Ex You invest $2,000 in a new business for one year that will return you $2,240. What is the rate of return?

Inflation affects the real interest rate.

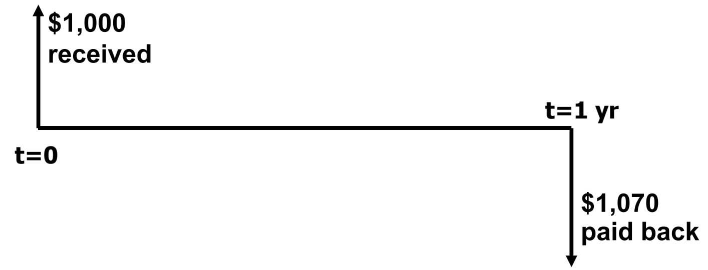
- Inflation = a decrease in the value of a currency
- Impacts of inflation:
  - Reduction in purchasing power
  - Increase in consumer prices
  - Increase in operating costs
  - Reduction in the real rate of return on investments

3.2) Economic Equivalence

- Equivalence Example
  - 80 miles per hour ~ 128 kilometers per hour
  - 80 mph is equivalent to 128 kph
  - Note, 80 is not equal to 128, but they are equivalent under two different measuring scales: miles/hour and kilometers/hour
- Economic Equivalence

  Different sums of money at different times can be *economically equivalent*, if we consider:
  - An interest rate
  - Time periods between the sums

Economic Equivalence



- $1,000 now is economically equivalent to $1,070 one year from now IF the interest rate is 7%/year.
- Or, if you are told that the interest rate is 7%, which is worth more, $1,000 now or $1,070 one year from now? Ans Two sums are economically equivalent, but not numerically equal.

3.3) Simple and Compound Interests

Simple Interest = an interest is calculated on the principal amount only

$$I = (P)(i)(n)$$

I = interest amount

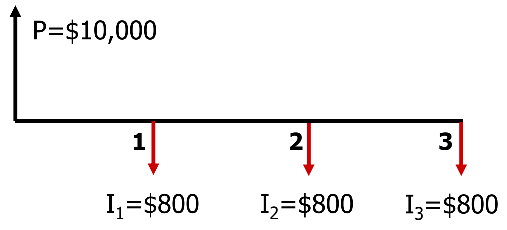
P = principal

i = interest rate

n = number of periods

Ex Calculate a simple interest for $10,000 borrowed for 3 years at 8% per year.

3.3.1) Simple Interest



Pay back $10,000 + $800 + $800 + $800

= $10,000 + $2,400 of interest, at the end of 3 years

The unpaid interest does not earn interest over the 3-year period

3.3.2) Compound Interest

- Compound interest is more common and widely-used in real-world applications.
- Compounding means to stop and compute interest owed at the end of each period and add it to the unpaid balance. This new unpaid balance is then used to calculate interest for the subsequent period.
- In compounding environment, accrued interest (owed but not yet paid) "earns interest."

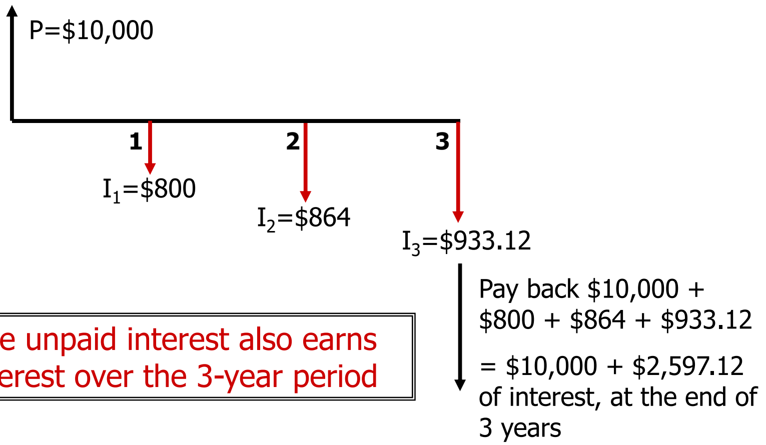
Compound Interest Example

Ex Calculate an interest for $10,000 borrowed for 3 years at 8% per year compounded annually.

Beginning: $P_0 = 10,000$

- $I_1 = (P_0)(i) = (10,000)(0.08) = 800$ (owed but not paid)
  End of yr1: $P_1 = P_0 + I_1 = 10,000 + 800 = 10,800$
- $I_2 = (P_1)(i) = (10,800)(0.08) = 864$ (owed but not paid)
  End of yr2: $P_2 = P_1 + I_2 = 10,800 + 864 = 11,664$
- $I_3 = (P_2)(i) = (11,664)(0.08) = 933.12$ (owed, not paid)
  End of yr 3: $P_3 = P_2 + I_3 = 11,664 + 933.12 = 12,597.12$
  This is the loan payoff amount at the end of 3 years.
- Note how the interest amount is added to form a new principal sum, on which the next period's interest is calculated.

Compound Interest Example



The unpaid interest also earns interest over the 3-year period

3.4) Loan Payment Example

Ex Suppose $10,000 is borrowed and will be paid back in 5 years, at an interest of 10% per year, compounded annually. Study the following three common payment schedules:

- Plan 1: Pay off the loan and interest at the end of five years
- Plan 2: Pay 20% of the principal back yearly, along with the current year's interest
- Plan 3: Make an equal payment every year for five years

Plan 1: Pay off the loan and interest at the end of five years

End of Year	Interest for the Year	Total Owed at End of Year	End-of-Year Payment	Remaining Unpaid Balance
0				$10,000.00
1	$1,000.00	$11,000.00	$0.00	$11,000.00
2	$1,100.00	$12,100.00	$0.00	$12,100.00
3	$1,210.00	$13,310.00	$0.00	$13,310.00
4	$1,331.00	$14,641.00	$0.00	$14,641.00
5	$1,464.10	$16,105.10	$16,105.10	$0.00
Total			$16,105.10	

Plan 2: Pay 20% of the principal back yearly, along with the current year's interest

End of Year	Interest for the Year	Total Owed at End of Year	End-of-Year Payment	Remaining Unpaid Balance
0				$10,000.00
1	$1,000.00	$11,000.00	$3,000.00	$8,000.00
2	$800.00	$8,800.00	$2,800.00	$6,000.00
3	$600.00	$6,600.00	$2,600.00	$4,000.00
4	$400.00	$4,400.00	$2,400.00	$2,000.00
5	$200.00	$2,200.00	$2,200.00	$0.00
Total			$13,000.00	

Plan 3: Make an equal payment every year for five years

End of Year	Interest for the Year	Total Owed at End of Year	End-of-Year Payment	Remaining Unpaid Balance
0				$10,000.00
1	$1,000.00	$11,000.00	$2,637.97	$8,362.03
2	$836.20	$9,198.23	$2,637.97	$6,560.25
3	$656.03	$7,216.28	$2,637.97	$4,578.30
4	$457.83	$5,036.13	$2,637.97	$2,398.16
5	$239.82	$2,637.97	$2,637.97	$0.00
Total			$13,189.87	

4) Cash Flow Symbols and Cash Flow Diagram

- P = a single amount or the value of money at present (at time 0) or earlier periods

  P is also referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC).

- F = a single amount or the value of money in the future (e.g. at the end of project) or later periods

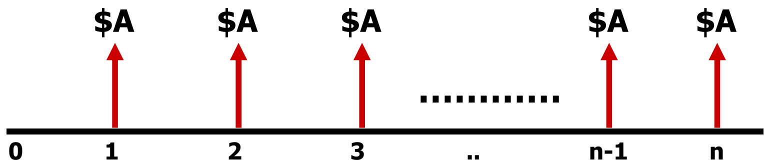
  F is also referred to as future worth (FW) or future value (FV).

4.1) Cash Flow Terms and Symbols

- A = a series of consecutive, equal, end-of-period amounts of money.

A is also called annual value (AV), annual worth (AW) or equivalent uniform annual worth (EUAW).

Note that A always represents a uniform amount (i.e., the same amount each period) that extends through consecutive periods.



Cash Flow Terms and Symbols

- i = interest rate or rate of return per time period
  - Generally assumed to be a compound rate
  - Expressed in percent per interest period. E.g., 10% per year, 1.5% per month
- n = number of periods

❖ Engineering Economics explore the relationship among these five parameters: P, F, A, i and n.

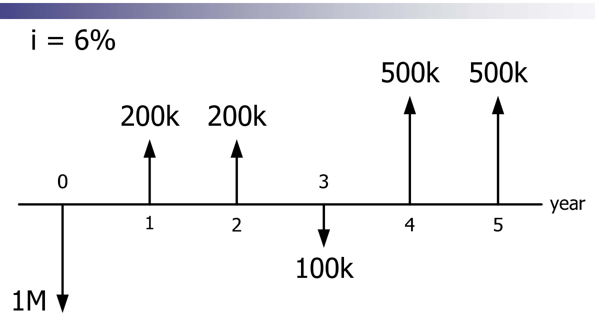
Excel Financial Functions (for reference)

- To find the present value P:
  PV(i%,n,A,F)
- To find the future value F:
  FV(i%,n,A,P)
- To find the equal, periodic value A:
  PMT(i%,n,P,F)
- To find the number of periods n:
  NPER(i%,A,P,F)
- To find the compound interest rate i:
  RATE(n,A,P,F)
  IRR(first_ cell:last_ cell)
- To find the present value P of any series:
  NPV(i%,second_cell:last_cell) + first cell

4.2) Cash Flow Diagram

- Graphical technique for representing a problem dealing with cash flows and their timing
- Types of cash flows:
  - Cash inflow: money flowing into the firm
    E.g. Revenues, savings, salvage values, etc.
  - Cash outflow: money flowing out of the firm
    E.g. Cost of assets, labors, salaries, taxes paid, utilities, rents, interests, etc.
  - Net cash flow = cash inflows – cash outflows
- CF are assumed to be known with certainty or can be estimated.

Cash Flow Diagram



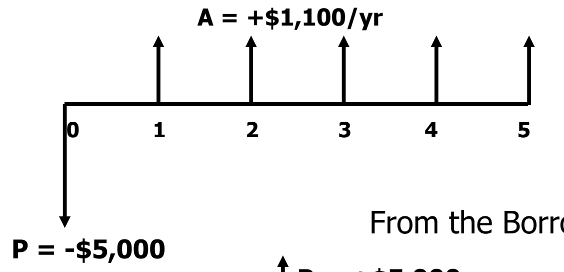
- For simplicity, all cash flows are assumed to occur at <u>the end of an interest period</u>.
- Positive cash flows are drawn upward.
- Negative cash flows are drawn downward.

Cash Flow Diagram: Perspective

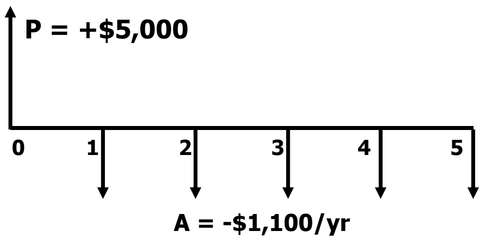
- Before drawing a cash flow diagram, one must decide upon the perspective of the problem
- For an example, in a borrowing situation:
  - Perspective 1: From the lender's view
  - Perspective 2: From the borrower's view
  - Impact upon the sign convention
- Assume $5,000 is borrowed and loan payments are $1,100 per year for five years. Draw a cash flow diagram.
  - Whose perspective, lender's or borrower's?
  - You must decide first.

Cash Flow Diagram: Two Perspectives

From the Lender's Perspective



From the Borrower's Perspective



Cash Flow Diagram Exercise

Ex You invest $1,000 every year in a fund that yields 15% per year, starting one year from now for 10 years. How much money will you have 20 years from now? Construct a cash flow diagram.

5) Minimum Attractive Rate of Return (MARR)

- Minimum Attractive Rate of Return (MARR) is a minimum rate that the firm requires for all accepted projects to meet or exceed.
- MARR is expressed as a percentage per year.
- Sometimes, MARR is called the hurdle rate.
- Why do we need to have MARR?
  - Capital needed for investment is not free.
  - For-profit firms expect to earn a return for their committed resources.

Determining MARR: Cost of Capital

- Individual's Cost of Capital: what is yours?
- Firm's Cost of Capital
  - Firm raises capital from two main sources:
    1. Debt: borrowing fund from outside the firm and paying interest, e.g. bank, bonds, etc.
    2. Equity: using the owner's funds, e.g. retained earning, share issuing, etc.
  - Financial models approximate the firm's weighted average cost of capital (WACC) for a given time
  - The projects must return at least the cost of the funds used in the project PLUS some additional return.

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Topic 2
Cash Flow Analysis and Valuation

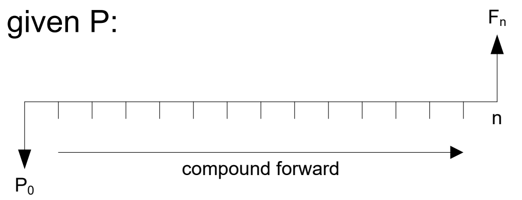
Roadmap

1) F/P and P/F Factors
2) P/A and A/P Factors
3) F/A and A/F Factors
4) Interest Factor Tables
5) Unknown Interest Rate and Number of Periods
6) Rule of 72
7) Application: Shifted Cash Flow Series and Combining Cash Flows

Section 1
F/P and P/F Factors

F/P: Compound Amount Factor

To find F, given P:



$$F_n = P(1+i)^n$$

where $(1+i)^n$ = (F/P, i%, n)

or

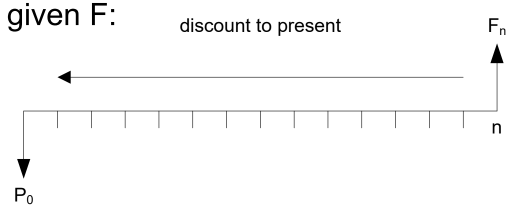
$$F_n = P(F/P, i\%, n)$$

Example: F/P Analysis

Ex Dora deposits $1000 at an interest rate of 2% per year. After four years, how much money will be in the account?

P/F: Present Worth Factor

To find P, given F:



$P = F_n(1+i)^{-n}$ or $P = F_n(P/F, i\%, n)$

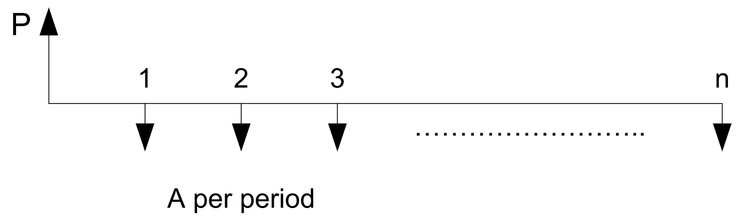
Example: P/F Analysis

Ex You are expected to receive $1,000,000 ten years from now. What is this amount worth today if an interest rate is 5% per year?

Section 2
P/A and A/P Factors

P/A: Uniform Series Present Worth Factor

To find P, given A:



A = equal, end-of-period cash flows, starting from t = 1 to t = n

P = ? (What is an equivalent present value?)

P/A Formula Derivation

P/A Formula

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \quad \text{for } i \neq 0$$

Ex You expect to receive $10,000 per year for four years, starting one year from now. What is an equivalent present value if an interest rate is 5% per year?

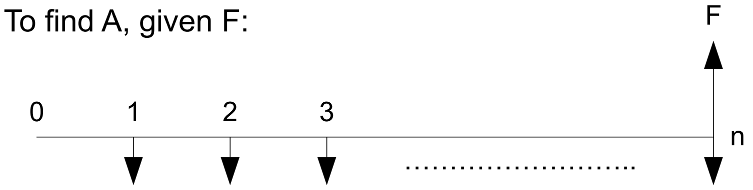
A/P: Capital Recovery Factor

Ex $10,000 is borrowed and will be repaid with equal end-of-year payments over the next five years. At an interest rate of 10% per year, what is the amount of each payment?

Section 3
F/A and A/F Factors

A/F: Sinking Fund Factor

To find A, given F:



Recall :

$$P = F\left[\frac{1}{(1+i)^n}\right]$$

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

(A/F, i%, n)

Example: A/F Analysis

Ex Ben wants to accumulate $1,000,000 for his retirement 40 years from now. How much does he need to save up each year in an account that yields 3% per year?

F/A: Series Compound Amount Factor

Ex If Joe is able to put away $12,000 per year in an investment fund that returns 6% per year. What will he have at the end of 30 years?

Section 4
Interest Factor Tables

All Engineering Economics textbooks provide interest factor tables, usually at the end of the text.

5.00%	Single Cash Flows		Uniform Cash Flow Series			
n	F/P	P/F	A/P	P/A	A/F	F/A
1	1.05000	0.95238	1.05000	0.95238	1.00000	1.00000
2	1.10250	0.90703	0.53780	1.85941	0.48780	2.05000
3	1.15763	0.86384	0.36721	2.72325	0.31721	3.15250
4	1.21551	0.82270	0.28201	3.54595	0.23201	4.31013
5	1.27628	0.78353	0.23097	4.32948	0.18097	5.52563
6	1.34010	0.74622	0.19702	5.07569	0.14702	6.80191
7	1.40710	0.71068	0.17282	5.78637	0.12282	8.14201
8	1.47746	0.67684	0.15472	6.46321	0.10472	9.54911
9	1.55133	0.64461	0.14069	7.10782	0.09069	11.02656
10	1.62889	0.61391	0.12950	7.72173	0.07950	12.57789

Using Interpolation to Estimate an Interest Factor

Ex Use interpolation to estimate the value of F/A factor for an interest of 5.2% and n = 20

(F/A, 5%, 20) =

(F/A, 6%, 20) =

i	F/A
5%	33.0660
5.2%	?
6%	36.7856

Interest Factor Interpolation Example (cont.)

$$\frac{5.2-5}{6-5} = \frac{x-33.0660}{36.7856-33.0660}$$

$$0.2 = \frac{x-33.0660}{3.7196}$$

X= 33.0660 + (0.2)(3.7196)

= 33.80992

Using formula : (F/A, 5.2%, 20) =

Some approximation error because an interest factor does not have a linear relationship with an interest rate.

Section 5
Unknown Interest Rate and Number of Periods

Finding Unknown Interest Rate (i)

Ex An investment requires an upfront capital of $20,000 and will pay back $35,000 four years from now. What interest rate will equate the two cash flows?

Finding Unknown Number of Periods (n)

Ex How long will it take for the money to triple its value if an interest rate is 8%?

Section 6
Rule of 72

Rule of 72

Rule of 72 states that .....

An approximate time (n) for an investment to double in value, given an interest rate (i) is:

$$n = 72/i$$

Rule of 72 Example

Ex What will an interest rate have to be in order for an investment to double its value in ten years? Calculate using factor formula and Rule of 72; compare the results.

Formula:

Rule of 72:

Rule of 72

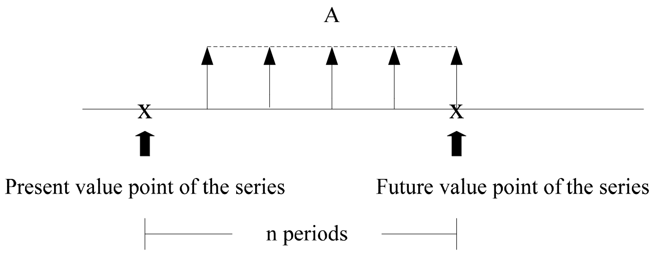
N	2	3	4	5	10	20	30	40
i from $2P = P(1+i)^n$	41.42%	25.99%	18.92%	14.87%	7.18%	3.53%	2.34%	1.75%
i from $n = 72/i$	36.00%	24.00%	18.00%	14.40%	7.20%	3.60%	2.40%	1.80%

The approximation is reasonably close for i <= 25% or 3 <= t <= 40 years.

Section 7 Application: Shifted Cash Flow Series and Combining Cash Flows

Shifted Uniform Cash Flow Series

- <u>Shifted</u> series are those whose <u>present value point</u> is <u>not at t = 0.</u>
- Uniform series: PV point is always one period to the left of the first cash flow.



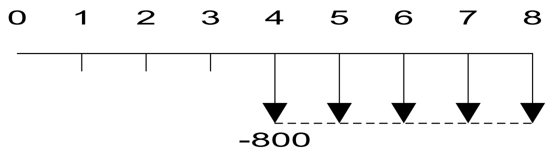
Shifted Uniform Cash Flow Series

To deal with combination of single cash flows and/or shifted series, here are the steps:

1. Draw a cash flow diagram.
2. Locate a PV point and a FV point of the series.
3. Write the time value of money equivalence relationship.
4. Substitute the factor values and solve.

Example: Shifted Uniform Cash Flow Series

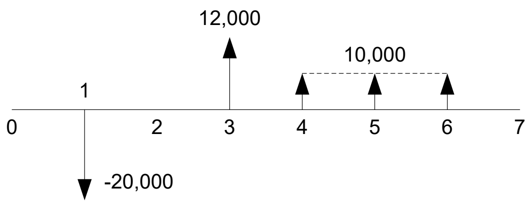
<u>Ex</u>



i = 3%, P_0 = ?, F_8 = ?

Example: Combination of Shifted Uniform Series and Single Cash Flows

<u>Ex</u>



i = 6%, P_0 = ?, F_7 = ?

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Topic 3
Nominal and Effective Interest Rates

Roadmap

1) Nominal vs. Effective Interest Rates

- Interest rates can be quoted in many ways:
  - 1.5% per month
  - 3% per quarter
  - 10% (per what?)
  - 12% per year, compounded quarterly
- What do they mean? How do they compare?
- Two types of interest rate quotation:
  1) Nominal Interest Rate
  2) Effective Interest Rate

Nominal Interest Rate

- Nominal interest rate = r
- A nominal interest rate is an interest rate that does not include any consideration of compounding

  r = (interest rate per period)(no. of periods)
- Example:
  - 1.5% per month for 24 months

    = (1.5%)(24) = 36% per 24 months
  - 2% per month for 6 months

Effective Interest Rate

- Effective interest rate = i
- An effective interest rate is a true, periodic interest rate (applied for a stated period)
- An effective interest for a year is called "Effective Annual Interest Rate" (EAIR)
- EAIR = nominal rate + frequency of compounding (e.g., 12% per year, compounded quarterly)

Nominal vs. Effective Interest Rates

- Nominal Interest Rate:
  - Format: "r% per time period"
  - E.g., 6% per six months
- Effective Interest Rate:
  - Format: "r% per time period, compounded m times a year"
  - E.g., 12% per year, compounded monthly
- The real world applications generally use an effective interest rate.

2) Effective Rate Per Compounding Period

Let

r = a nominal rate per time period t

m = no. of times that an interest is compounded within the time period t

i = an effective rate per compounding period

$$i_{\text{effective per CP}} = \frac{r}{m}$$

Example: Effective Rate Per Compounding Period

Ex 12% per year, compounded quarterly: what is an effective quarterly rate?

Quarter 1	Quarter 2	Quarter 3	Quarter 4
3%	3%	3%	3%

The true (effective) rate per quarter is 3% (because there is no compounding within a quarter).

Example: Effective Rate Per Compounding Period

Ex Find an effective rate per compounding period for the followings:

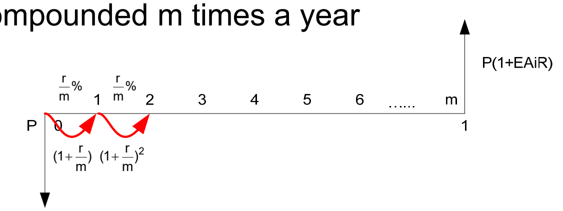
- 15% per year, compounded monthly
- 8% per year, compounded semiannually
- 6% per quarter, compounded monthly

3) Effective Annual Interest Rate (EAIR)

- EAIR is a true, effective, annual interest rate (already taken in the compounding frequency)
- One year can be divided into many compounding periods:
  - Compounded annually (c.a., m = 1)
  - Compounded semiannually (c.s.a., m = 2)
  - Compounded quarterly (c.q., m = 4)
  - Compounded monthly (c.m., m = 12)
  - Compounded weekly (c.w., m = 52)
  - Compounded daily (c.d., m = 365)

EAIR Derivation

r%, compounded m times a year



One year is divided into m compounding periods.

$$EAIR = (1+\frac{r}{m})^m - 1$$

effective rate per compounding period ($\frac{r}{m}$)

Example: EAIR

Ex Given an interest rate of 12%, compounded monthly, what is the true annual interest rate?

Note: a nominal rate of 12% gives two effective rates:

- effective monthly rate = 1% per month
- effective annual rate = 12.68% per year

EAIR for 12% at Various Compounding Periods

Compounding Frequency	No. of Compounding Periods	EAIR Calculation	EAIR
Annual	1	$(1+0.12/1)^1 - 1$	12%
Semiannual	2	$(1+0.12/2)^2 - 1$	12.36%
Quarterly	4		
Bi-monthly	6	$(1+0.12/6)^6 - 1$	12.61624%
Monthly	12	$(1+0.12/12)^{12} - 1$	12.68250%
Weekly	52	$(1+0.12/52)^{52} - 1$	12.73410%
Daily	365		
Hourly	8760	$(1+0.12/8760)^{8760} - 1$	12.74959%

Default: Without specification, an interest rate is **per year**, and the compounding frequency is **annually**. (e.g. 12% = 12% per year, compounded annually)

4) Effective Rate Per Payment Period

- CP = compounding period (period between compounding)
- PP = payment period (period between cash flow payments)
- In practice, the payment period may not be the same as the compounding period. E.g., an interest is compounded monthly, while loan payments are made every six months
- Key in finding an economic equivalence:

Find an effective rate per payment period

4) Effective Rate Per Payment Period

Ex What is an effective rate per quarter if r = 12%, c.m.?

4) Effective Rate Per Payment Period

$$i_{\text{eff per PP}} = (1 + i_{\text{eff per CP}})^n - 1$$

$i_{\text{eff per PP}}$ = effective rate per payment period
$i_{\text{eff per CP}}$ = effective rate per compounding period
n = the number of times an interest is compounded in one specified period (or payment period)

Ex What is an effective rate per six months if r = 18%, c.m. ?

Interest Rate Comparisons

To compare interest rates, find their effective rates for the same period, e.g. EAIR or an effective rate per PP.

Ex Payments on a loan are made every six months, and three charging plans are available for consideration.

Plan 1: 16% per year, compounded quarterly

Plan 2: 15% per year, compounded monthly

Plan 3: 8.2% per 6 mo., compounded semiannually

Which plan is the cheapest?

Example: Plan 1

Plan 1: 16% per year, compounded quarterly

Effective rate for 6 months =

EAIR =

Example: Plan 2

Plan 2: 15% per year, compounded monthly

Effective rate for 6 months =

EAIR =

Example: Plan 3

Plan 3: 8.2% per 6 mo, compounded semiannually

Effective rate for 6 months =

EAIR =

Example: Comparison of Three Plans

Plan	6-month rate	EAIR
1	8.16%	16.986%
2	7.74%	16.075%
3	8.2%	17.07%

- Plan 2 yields the lowest effective rate, semiannually or annually.
- To compare interest rates, compare an effective rate per payment period or EAIR.

5) Applying Effective Rate to Single Cash Flows

- When CP ≠ PP, we are only interested in the case where CP ≤ PP.
- For CP > PP, it depends on the convention on how to approximate. (out of scope for this class)
- For single cash flows, when CP ≤ PP, there are two approaches:
  1) Adjust the time scale to match the compounding period (CP).
  2) Find EAIR and always count time in years.

Example: Single Cash Flows

Ex At an interest rate of 12%, c.q., $1000 is deposited now. How much will be an account three years from now?

Approach 1:

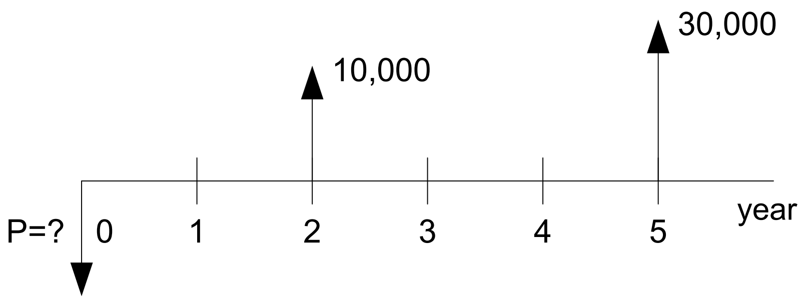
Example: Single Cash Flows (cont.)

Approach 2:

Note some minor difference due to round-off errors.

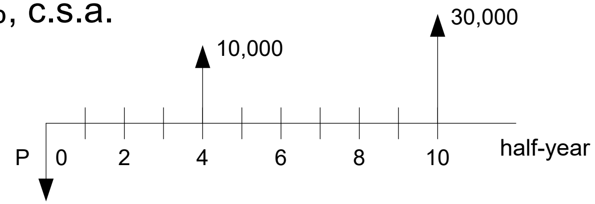
Another Example: Single Cash Flows

Ex r = 10%, c.s.a.

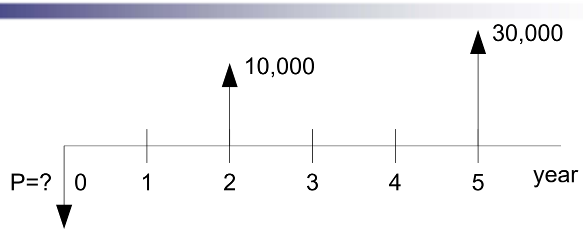


Approach 1: Count time in 6-month period

r = 10%, c.s.a.



Approach 2: Count time in years, find EAIR



6) Applying Effective Rate to Cash Flow Series

- The only correct method is to find an effective rate per payment period.
- A payment period is a period between two cash flows in the series.
- E.g., payments are made on a loan every six months -> find an effective rate per six months
- This is because our series formulas are based on:
  - an interest rate that is an effective rate per PP.
  - cash flows which occur exactly once every period from period 1 to n

Example: Cash Flow Series

Ex Payments of $1000 are made on a loan every six months for ten years. At 6%/yr, compounded quarterly, what is the present value of these payments?

7) Continuous Compounding

- Continuous compounding is when the time between compounding approaches 0, or the number of compounding periods (in a year or any finite time) approaches infinity.
- Recall:

$$EAIR = (1+\frac{r}{m})^m - 1$$

- For continuous compounding, $m \rightarrow \infty$

Derivation of Expression for Continuous Compounding

$$i = e^r - 1$$

r = a nominal interest rate that is compounded continuously

Example: Continuous Compounding

Ex What is the true, effective, annual interest rate if r = 12% per year, compounded continuously?

Note This is the maximum EAIR for a nominal rate of 12% (among all compounding frequencies).

Ex For r = 15% per year, c.c.

a) An effective annual rate:

b) An effective monthly rate:

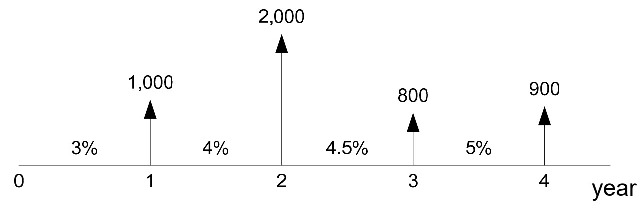
Example: Continuous Compounding

Ex A fund has an effective return (interest rate) of 20% when it is compounded continuously. What is the nominal rate?

8) Varying Interest Rates Over Time

- In the real world, an interest rate may vary over time
- Approach: do a period-by-period analysis

Ex



P = ?

Example: Varying Interest Rates

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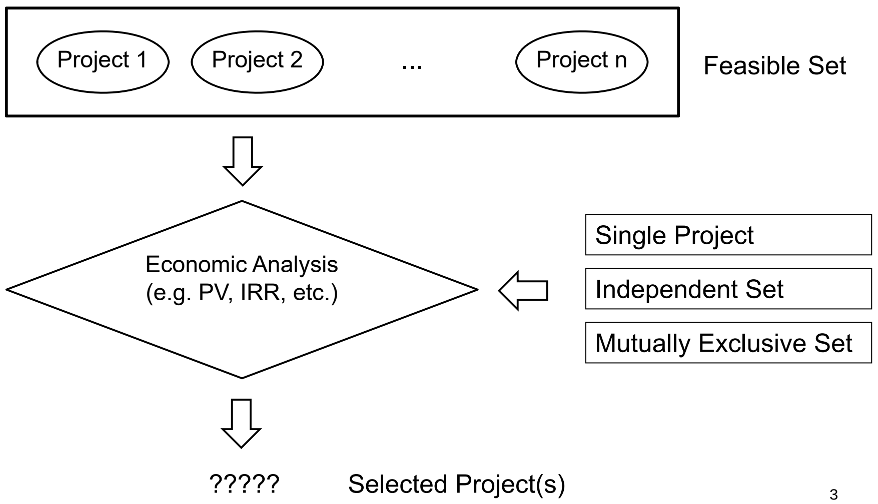
Topic 4
Present Value Analysis

Roadmap

Characterizing Alternatives



1) Present Value Method

- Present value (PV) is an <u>equivalent value</u> of all future cash flows during a project life.
- An interest rate used to discount the future cash flows is generally the firm's MARR (or higher).
- MARR represents the firm's "opportunity cost," – alternative use of the firm's fund, i.e. other investments.
- Present value transforms the future cash flows into an equivalent dollar NOW.

Present value is a function of an assumed interest rate.

- If the cash flows contain a mixture of positive and negative cash flows, calculate
  - PV (i%) of positive cash flows [PV(income/revenue)]
  - PV (i%) of negative cash flows [PV(expense)]
  - Add the results [PV(total) or Net Present Value: NPV]
- The results can be added because they are equivalent values at the same point in time.
- Interpretation:
  - PV(total) > 0, PV(income/revenue) > PV(expense)
  - PV(total) < 0, PV(income/revenue) < PV(expense)
  - PV(total) = 0, PV(income/revenue) = PV(expense)

2) Single Project Decision

- No competing projects
- The firm has two alternatives: accept or reject the project.
- Criteria: evaluate the single project's present value at the firm's MARR.
  - PV (MARR) > 0, accept the project
  - PV (MARR) < 0, reject the project
  - PV (MARR) = 0, indifferent
- Note an implicit comparison with the firm's opportunity cost.

Example: Single Project Decision

Ex A project requires investing $1,000,000 now and will generate an income of $200,000 per year over the next five years, starting one year from now, with a salvage value of $300,000 at the end of year 5. At MARR of 12%, should the firm invest in this project?

3) Independent Project Decision

- For an independent set, more than one project can be selected.
- With a budget limitation, the problem is often formulated as 0-1 linear programming model (outside scope of this class).
- Without a budget limitation, decide on each project independently, similar to a single project decision – accept if PV(i) > 0 and reject otherwise.

4) Mutually Exclusive Projects with Equal Life

- Mutually exclusive projects compete with one another, and we only select one.
- To compare using the PV approach, the lives of all projects must be equal.
- For equal-life projects, calculate NPV(i) over their lives and select the project with the highest positive NPV. For a cost project (must-do), choose the project with the lowest PV of costs.
- Use the same discount rate (MARR or higher) for all projects.

Example: Equal-Life Projects

Ex Suppose A, B and C machines possess the same capability and will generate sufficiently large revenue. At MARR of 9%, which one should be selected based on PV analysis?

	Manual (A)	**Semi-automatic (B)**	**Fully automatic (C)**
First Cost	-10,000	-15,000	-18,000
Annual Operating Cost (AOC)	-4,000	-2,000	-1,500
Salvage Value	+2,000	+3,000	+4,000
Life	4	4	4

Example: Equal-Life Projects (cont.)

Example: Equal-Life Projects (cont.)

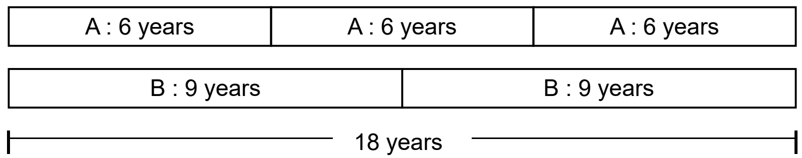
Alternative B (semi-automatic machine) is the cheapest because the PV of its cost is minimum.

5) Mutually Exclusive Projects with Different Lives

- PV of alternatives must be compared over the same time horizon.
- If the alternatives' lives are not equal, there are two approaches:
  1) Least Common Multiple (LCM) = compare the alternatives over LCM of their lives
  2) Study Period = compare alternatives over a specified study period

PV Using the LCM Approach

Ex Project A = 6 years, Project B = 9 years, LCM = 18 years



To use the LCM approach, the following assumptions are needed:

1) The service (provided by alternatives) is needed for at least LCM years.
2) The alternatives' life cycle can be repeated.
3) Cash flow estimates in every life cycle are the same.

Example: PV Using LCM Approach

Ex Two types of equipment with the same capability are available at the following costs. At MARR = 12%, which one is better based on PV analysis?

	A	B
First Cost	-10,000	-19,000
Annual Operating Cost (AOC)	-2,000	-1,000
Salvage Value	+1,000	+2,000
Life	4	6

Example: PV Using LCM Approach (cont.)

Without a specified period of study, use LCM.
LCM of 4 and 6 =
∴ Calculate PV over years by repeating the cycles and compare.

Example: PV Using LCM Approach (cont.)

PV Using the Study Period Approach

- In some situations, a firm may only be interested in the cash flows over a specified period.
- Certain assumptions on the cash flows must be made according to the problem setting.
- It may simplify the calculation, compared to the tedious numerical computation required in the LCM approach.

Example: PV Using the Study Period Approach

Ex Using the data on two types of equipment in the previous example, if the company only needs an equipment for 6 years and the salvage value is the same regardless of when it is sold, which equipment should the company select?

Example: PV Using the Study Period Approach (cont.)

6) Future Value Analysis

- For some applications, comparing the future value can be more convenient and provides a direct interpretation.
- Example applications:
  - Wealth maximization
  - Projects that come online after a construction period, e.g., power generation plant, toll road, real estate development such as condo, etc.
- Methodology: calculate F_n (from the cash flows directly or from the PV)
- Decision criteria are similar to those of PV.

7) Capitalized Cost

- Capitalized cost is the present value of a project which lasts forever.
- Example projects include: roads, bridges, dams (assumed to have an infinite life).
- Assume that there are uniform, end-of-period cash flows of A that run to infinity, find PV (called capitalized cost: CC).

Capitalized Cost Derivation

Recall P/A factor:

Capitalized Cost, $n \rightarrow \infty$

$$\therefore \quad \boxed{P = CC = \frac{A}{i}}$$

Example: Capitalized Cost

Ex A wealthy family would like to establish a scholarship fund, where $10,000 can be withdrawn each year to forever to give as a scholarship. If the fund can generate a return of 10%, what must be the size of this fund?

Capitalized Cost: Complex Problems

- Some projects involve recurring (periodic and repeat) and non-recurring costs.
- Methodology:
  1. For non-recurring costs, calculate PV using time-value equivalence (find P)
  2. For recurring costs (that last forever),
     a) Convert recurring costs into annualized equivalent amount, A. [Using (A/F, i%, n)]
     b) Use CC = A/i to find the capitalized cost.
  3. Add PV of (1) and (2) together to find the total PV.

Example: Complex Capitalized Cost

Ex A community considers creating a small park on their common ground. The initial cost of landscaping and buying plants and trees is $500,000 now. An annual maintenance cost is expected to be $30,000/year. The park also needs major reconditioning every three years at a cost of $100,000. Moreover, the community intends to purchase new plants and decorations every five years at a cost of $200,000. The park is expected to last forever. At 6% interest rate,

a) What is the capitalized cost of the project?
b) What is an equivalent annualized cost?

Example: Complex Capitalized Cost (cont.)

a)

Example: Complex Capitalized Cost (cont.)

Example: Complex Capitalized Cost (cont.)

∴ Total capitalized cost of project = PV(landscape) + CC_1 + CC_2 + CC_3

= 500,000 + 500,000 + 523,516.67 + 591,333.33

= 2,114,850

Example: Complex Capitalized Cost (cont.)

b)

∴ An equivalent annualized cost is $126,891/year.

In other words, the capitalized cost (PV) of the project is $2,114,850. This is equivalent to paying $126,891 every year to forever.

8) Payback Period Analysis

- Payback period estimates how long it takes to recover the initial investment.
- Generally, a shorter payback period is preferred to a longer payback period.
- Payback period is popular as a screening tool by managers.
- It should not be used to make the final decision because the cash flows after the payback period are ignored.

Payback Period Analysis

- Two forms of payback period:
  1) With 0% interest
  2) With an assumed interest rate
- A formal definition of the payback period (n_p):

$$0 = -P + \sum_{t=1}^{t=n_p} NCF_t (P/F, i\%, t).$$

Example: Payback Period with 0% Interest Rate

Ex For an investment with the below cash flows, how long does it take to recover the initial investment at 0% interest rate?

t	End of Year Cash Flow	Cumulative Cash Flow
0	-10,000	-10,000
1	-2,000	-12,000
2	+4,000	-8,000
3	+5,000	-3,000
4	+6,000	+3,000
5	+7,000	+10,000

⇦ Payback period is 3 < n < 4

Cash flows go from - from + between year 3 and 4

Example: Payback Period with 12% Interest Rate

t	End of Year Cash Flow (CF_t)	(P/F, 12%, t)	CF_t(P/F, 12%, t)	Cumulative Discounted Cash Flow
0	-10,000.00	1.00000	-10,000.00	-10,000.00
1	-2,000.00	0.89286	-1,785.72	-11,785.72
2	4,000.00	0.79719	3,188.76	-8,596.96
3	5,000.00	0.71178	3,558.90	-5,038.06
4	6,000.00	0.63552	3,813.12	-1,224.94
5	7,000.00	0.56743	3,972.01	2,747.07

- With 12% interest, the payback period is 4 < n < 5.
- For the same cash flows, note that the payback period is longer when an interest rate increases. Why?

End of Slide Set

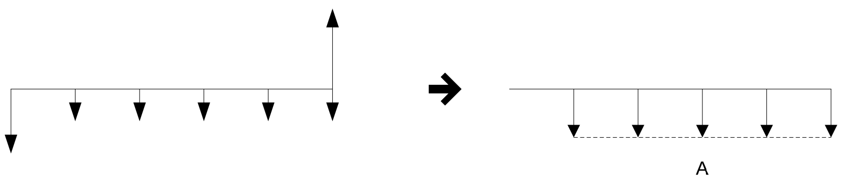
Topic 5
Annual Value Analysis

Roadmap

1) Annual Value (AV) Calculation
2) Analysis Using Annual Value
3) Capital Recovery (CR)
4) Alternative Selection Using Annual Value
5) Annual Value of Perpetual Investment

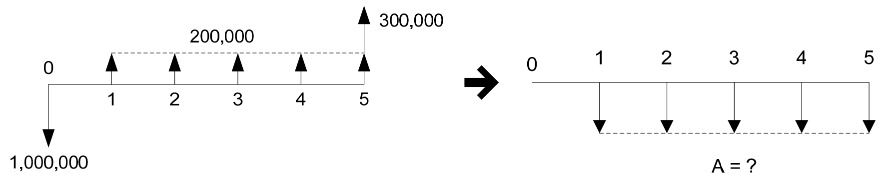
1) Annual Value (AV) Calculation

- Popular analysis technique among managers
- Talk about profit/revenue/cost in terms of $/year
- Calculation: Convert project cash flows into equal, end-of-period equivalence, A
  - A = P(A/P, i%, n)
  - A = F(A/F, i%, n)



Example: Annual Value Calculation

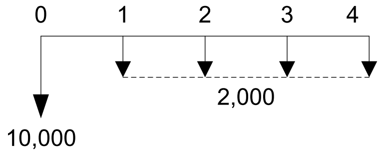
Ex A project requires investing $1,000,000 now and will generate an income of $200,000 per year over the next five years, starting one year from now, with a salvage value of $300,000 at the end of year 5. At MARR of 12%, find an annual value of the project.



AV of one cycle equals to AV of multiple cycles.

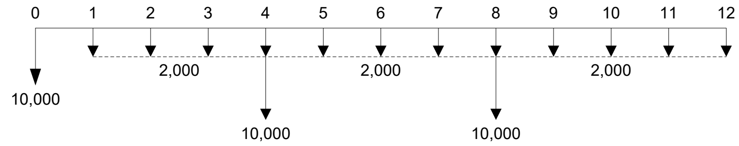
<u>Ex</u> Consider a project with an investment cost of $10,000 and AOC of $2,000/year for 4 years. At MARR = 12%, find AV of 1 cycle and 3 cycles.

AV of one cycle:



Example (cont.)

AV of three cycles:



Note that AV of 1 cycle is the same as AV of 3 cycles, except for a round-off error.

2) Analysis Using Annual Value

- If the alternatives have unequal lives, when comparing among alternatives, only need to evaluate <u>AV of one cycle</u> for each alternative.
- AV of one cycle is the same as AV of many cycles (as shown in the previous example).
- Thus, no need to find LCM and calculate over LCM years (as in PV analysis)!! Therefore, AV can be much simpler than PV.
- Assumptions (for unequal-life projects) are similar to PV:
  1) Service is needed forever (or at least LCM years).
  2) Cycles of cash flows can be repeated.
  3) Cash flows are the same in every life cycle.

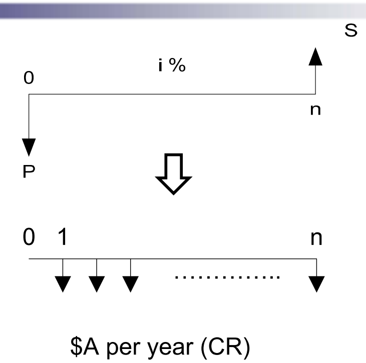
Applications of Annual Value Analysis

- Annual value analysis is applied in:
  - Asset Replacement
  - Breakeven Analysis
  - Build or Rent Decisions
  - etc.
- Similar to PV analysis, AV analysis requires:
  - A discount rate (MARR)
  - estimate of future cash flows
  - time period

3) Capital Recovery (CR)

- Capital recovery is an <u>equivalent annual cost of owning</u> a productive asset.
- Capital recovery is a function of
  - P = initial investment
  - S = estimated future salvage value
  - n = estimated life of the asset
  - i = an interest rate or a discount rate
- *Operating and maintenance costs* are additional annual costs but they are <u>not</u> part of the capital recovery.

Capital Recovery (CR)



CR = P(A/P, i, n) - S(A/F, i, n)

(investment cost = +, sv = -
Or, investment cost = -, sv = +)

Capital Recovery Cost Calculation

- <u>Method 1:</u> Compute AV of the initial investment cost less AV of the salvage value.

  CR(i%) = P(A/P, i, n) - S(A/F, i, n)

- <u>Method 2:</u> Compute AV of the difference between the initial investment cost and the salvage value. Then add an interest that the salvage value would generate each year S(i), since it is actually sold in year n.

  CR(i%) = (P-S)(A/P, i, n) + S(i)

Example: Capital Recovery

<u>Ex</u> You are buying a car now at $30,000 and it will sold 3 years from now at $20,000. At 6%, what is the capital recovery of this car?

The cost of owning the car (without driving, maintenance, insurance, etc.) is $4,941 per year.

4) Alternative Selection Using Annual Value

- Needs an agreed-upon discount rate
- Criteria for AV are similar to those of PV.
- Single alternative or independent projects without a budget limit:
  - Accept if AV(i) is positive and reject otherwise
- Mutually exclusive projects:
  - Select an alternative with the lowest AV(i) of costs or the highest AV(i) of revenue or profit

Example: Alternative Selection Using AV

Ex Two types of equipment with the same capability are available at the following costs. At MARR = 12%, what are the capital recovery of A and B? Which one is cheaper?

	A	B
First Cost	-10,000	-19,000
Annual Operating Cost (AOC)	-2,000	-1,000
Salvage Value	+1,000	+2,000
Life	4	6

Example: Alternative Selection Using AV (cont.)

∴ Select A ∵ lower AV cost (note the same answer as PV analysis)

5) Annual Value of Perpetual Investment

- Perpetual investment is an investment without a finite cycle (last forever).
- Recall from capitalized cost analysis (in Ch. 5):

$$P = \frac{A}{i} \quad \Longrightarrow \quad \boxed{A = P(i)}$$

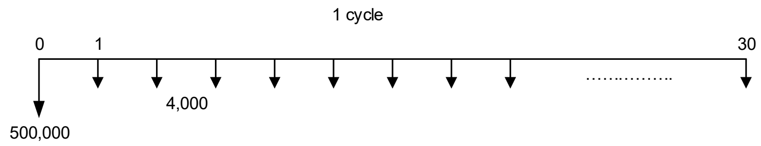
Example: AV of Perpetual Investment

Ex Two alternatives of securing a location for business are presented. The first one is to buy at $1,000,000. The second one is to rent which involves paying $500,000 every 30 years starting now plus an annual cost of $4,000. Assuming the business lasts forever and MARR is 10%, which one is better?

Alternative 1: To Buy

Example: AV of Perpetual Investment (cont.)

Alternative 2: To Rent



∴ Renting is cheaper.

End of Slide Set

Topic 6
Rate of Return Analysis: Single Alternative

Roadmap

1) Rate of Return Calculation
2) Single-Alternative Decision Using ROR
3) Cautions When Using ROR
4) Bonds: Present Value and ROR

1) What is the rate of return (ROR)?

- A popular measure of an investment project, besides the present value method.
- The rate of return (ROR) is the interest rate that equates the value of the future cash flows (revenue) to the initial investment.
- Given a stream of cash flows, a rate of return (i*) is implied by these cash flows.

Rate of Return Calculation

Find i* such that

1) PV (negative cash flows, at i*)
= PV (positive cash flows, at i*)

Or,

PV (all cash flows with signs, at i*) = 0

2) AV (neg. CF, at i*) = AV (pos. CF, at i*)

Or,

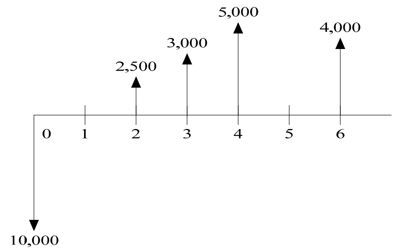
AV (all cash flows with signs, at i*) = 0

Example: ROR Calculation

Ex A project requires an initial investment of $100,000 and will generate an annual income of $27,741 over the next five years, starting one year from now. What is the rate of return on the project?

Example: ROR Calculation

Ex What is the rate of return on the following project?



Can you solve for i* directly?

If you cannot solve for i* directly, use trial and error approach or Excel.

Determining ROR by Trial and Error

Procedure:

1) Guess a rate and plug it in an equation (PV = 0)
2) Adjust accordingly
   - If PV is negative, your guess for i* is too
   - If PV is positive, your guess for i* is too
3) Bracket (one i* yields a positive PV, one i* yields a negative PV)
4) Interpolate for i* that will make PV = 0

Otherwise, use Excel:

RATE(n, A, P, F) or IRR(firstCell:lastCell, guess)

2) Single-Alternative Decision Using ROR

- After obtaining i* for the project cash flows,
  - If i* > MARR, accept the project.
  - If i* < MARR, reject the project.
  - If i* = MARR, indifferent.
- Why?

3) Cautions When Using ROR

1) Multiple i*
- Since PV(i*) = 0 is a non-linear equation, there could potentially be multiple i* that solve the equation.
- Ex

Year	Cash Flow
0	3,600
1	-6,000
2	-10,000
3	15,000
4	1,000

use Excel → PV(37.4%) = 0 and PV(96.4%) = 0

- Which i* should be used in reality?

Cautions When Using ROR (cont.)

2) Reinvestment Assumptions
- PV and AV assume reinvestment at MARR rate.
- ROR assumes reinvestment at i* rate.

3) Computational Difficulty
- ROR can be computationally difficult for the real-world cash flows.
- Use Excel (numerical method) for analysis.

4) For multiple alternatives, an "incremental analysis" is required. (Topic 7)

Summary on Usage

- Mathematically, -100% < i* < ∞

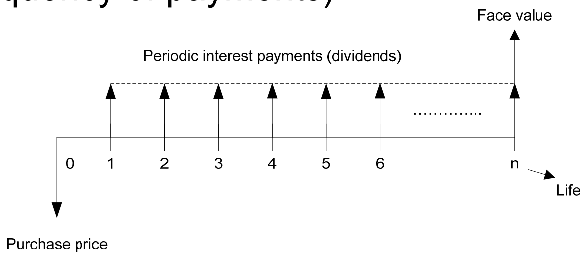
  i* = -100% signals a total loss of capital.
- ROR is popular among managers.
- ROR could be computationally much more difficult than PV/AV.
- To decide on projects, you can use PV/AV (arguably simpler) or ROR.
- When is ROR particularly useful?

4) Bonds

- Bonds are an instrument to raise debt capital to finance operations.
- Bonds = "iou" document
- Bonds pay a stated rate of interest to the bond holder for a specified period.
- Various types of bonds:
  - treasury bonds (federal government: bills = less than 1 year, notes = 2-10 years, bonds = 10-30 years)
  - municipal bonds (local government)
  - mortgage bonds (corporation, backed by asset)
  - debenture bonds (corporation not backed by asset)

Bond Parameters and Cash Flows

1. Face Value ($100, $1000, $5000, etc.)
2. Life of bond (years)
3. Coupon rate (nominal interest rate and frequency of payments)



Periodic interest payments = periodic interest rate x face value

Converting Bond Description into Cash Flows

Ex Draw a cash flow diagram of the bond, "$1000, 10-year bond with an interest rate of 6%, paid quarterly"

Present Value of Bonds

Ex Consider a $5000, 20-year bond which pays 5% per year, paid semiannually. If an investor requires a rate of return of 6% per year, c.q., what is the purchase price that the investor is willing to pay?

Present Value of Bonds (cont.)

Key
1) Think of bond's interest as a dividend (periodic payments, $).
2) Discount the bond cash flows at the investor's required interest rate or the market interest rate.

ROR of Bonds

Ex $1000 bond has a life of 10 years and pays 7% per year, paid semiannually. Immediately after four payments (two years), you buy this bond from the market at $1200. What is the rate of return on the bond that you get?

ROR of Bonds (cont.)

End of Slide Set

Topic 7
Rate of Return Analysis: Multiple Alternatives

Roadmap

1) Why Incremental Analysis?
2) Incremental Analysis Methodology
3) Interpretation of i* of the Incremental Investment
4) Incremental Analysis for Multiple Mutually-Exclusive Alternatives

1) Why Incremental Analysis?

- Earlier, we can use PV or AV to compare multiple alternatives. Note that, to use PV, projects must have equal lives (otherwise, need LCM or a study period).
- Now we will use ROR to decide among multiple projects.
- Ex Imagine two investments:

  A: Invest $100, pays $180 one year from now

  B: Invest $1,000,000, pays $1,500,000 one year from now

  Which one would you choose?

Why Incremental Analysis? (cont.)

Why Incremental Analysis? (cont.)

Assumption: We cannot do 10,000 pieces of A.

Why Incremental Analysis? (cont.)

- To use ROR to decide among multiple alternatives, we <u>cannot</u> just compare ROR of the projects.
- PV and ROR have different reinvestment (investment of extra cash flows) assumptions:
  - PV assumes reinvestment at MARR.
  - ROR assumes reinvestment at i*.
- For ROR to rank alternatives the same correct way as PV, we have to use the <u>incremental analysis</u>.

2) Incremental Analysis Methodology

Given two <u>mutually exclusive</u> alternatives,

1) Calculate ROR_A and ROR_B,
   - If any ROR < MARR, reject that project.
   - If ROR_A and ROR_B >= MARR, both projects are good candidates. Continue to the next step.
2) Call the project with a lower <u>initial</u> investment (investment at t=0) "A" and the more expensive one "B."
3) Calculate the incremental cash flows, Δ(B-A).
4) Compute $ROR_{(B-A)}$,
   - If $ROR_{(B-A)}$ > MARR, the incremental investment is worth it. Pick B.
   - If $ROR_{(B-A)}$ < MARR, the incremental investment is <u>not</u> worth it. Pick A.

Example: Incremental Analysis

<u>Ex</u> Project A and B are mutually exclusive. Use ROR to select the best alternative, when MARR = 10%.

t	A	B
0	$-20,000.00	$-35,000.00
1	$ 7,000.00	$ 12,000.00
2	$ 7,000.00	$ 12,000.00
3	$ 7,000.00	$ 12,000.00
4	$ 7,000.00	$ 12,000.00
5	$ 7,000.00	$ 12,000.00
ROR	22%	21%
PV(10%)	$ 6,535.51	$ 10,489.44

ROR_A > MARR and ROR_B > MARR.

Thus, A and B are good candidates to invest in.

Example: Incremental Analysis (cont.)

t	A	B	B-A
0	$-20,000.00	$-35,000.00	$-15,000.00
1	$ 7,000.00	$ 12,000.00	$ 5,000.00
2	$ 7,000.00	$ 12,000.00	$ 5,000.00
3	$ 7,000.00	$ 12,000.00	$ 5,000.00
4	$ 7,000.00	$ 12,000.00	$ 5,000.00
5	$ 7,000.00	$ 12,000.00	$ 5,000.00
ROR	22%	21%	20%
PV(10%)	$ 6,535.51	$ 10,489.44	$ 3,953.93

ROR_{B-A} > 10%

ROR_A > MARR means that A is worth investing.
ROR_{B-A} > MARR means that B-A is worth investing.
Thus, B is worth investing. Select B.

Another Example: Incremental Analysis

t	A	B	B-A
0	$-20,000.00	$-35,000.00	$-15,000.00
1	$ 7,000.00	$ 10,500.00	$ 3,500.00
2	$ 7,000.00	$ 10,500.00	$ 3,500.00
3	$ 7,000.00	$ 10,500.00	$ 3,500.00
4	$ 7,000.00	$ 10,500.00	$ 3,500.00
5	$ 7,000.00	$ 10,500.00	$ 3,500.00
ROR	22%	15%	5%
PV(10%)	$ 6,535.51	$ 4,803.26	$ -1,732.25

ROR_{B-A} < 10%

ROR_A > MARR means that A is worth investing.
ROR_{B-A} < MARR means that B-A is not attractive.
Thus, select A.

Example: Incremental Analysis of Cost Projects

t	A	B	B-A
0	$-30,000.00	$-40,000.00	$-10,000.00
1	$ -4,000.00	$ -1,000.00	$ 3,000.00
2	$ -4,000.00	$ -1,000.00	$ 3,000.00
3	$ -4,000.00	$ -1,000.00	$ 3,000.00
4	$ -4,000.00	$ -1,000.00	$ 3,000.00
5	$ -4,000.00	$ -1,000.00	$ 3,000.00
ROR	N/A	N/A	15%
PV(10%)	$-45,163.15	$-43,790.79	$ 1,372.36

ROR_{B-A} > 10%
Select B.

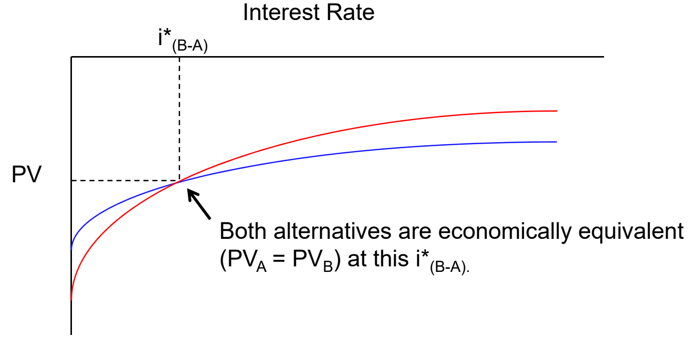
Note that for a required cost project, we may not be able to calculate ROR_A or ROR_B. However, we can still apply an incremental analysis to choose between A and B.

3) Interpretation of i* of the Incremental Investment (i^*_{B-A})

- i^*_{B-A} is the ROR of the additional investment to move from a cheaper project to a more costly project.
  - If i^*_{B-A} < MARR, the increment is not worth it, we should select the cheaper project.
  - If i^*_{B-A} > MARR, the increment is worth it, we should select the more costly project.
- i^*_{B-A} is also the interest rate at which two alternatives are economically equivalent. This special interest rate is called the breakeven interest rate or the Fisherian Intersection rate.

The Breakeven Interest Rate

From the last example



Usage of i^*_{B-A}

- If MARR < i^*_{B-A}, extra investment is justified. Select B.
- If MARR > i^*_{B-A}, extra investment is not justified. Select A.
- If MARR = i^*_{B-A}, both alternatives are economically equivalent.

4) Incremental Analysis for Multiple Mutually-Exclusive Alternatives

Key: Select the largest investment whose increment over another acceptable alternative is justified.

Generalized Process for Multiple Alternatives:

1) Calculate ROR or PV of all projects. Discard any project whose ROR < MARR or PV < 0.
2) Order the remaining (attractive) alternatives from the smallest initial investment to the largest initial investment, and call them: A, B, C, etc.

Incremental Analysis for Multiple Alternatives (cont.)

3) Compare the first pair (A and B). Call the cheaper one “Defender” and the more expensive one “Challenger.”
   a) Compute Δ(Challenger – Defender) cash flows.
   b) If $i^*_{Challenger-Defender}$ < MARR, Defender wins.
      If $i^*_{Challenger-Defender}$ >= MARR, Challenger wins.
      Or,
      If PV (Challenger – Defender) < 0, Defender wins.
      If PV (Challenger – Defender) >= 0, Challenger wins.
   c) Whoever is the winner becomes the Defender for the next round.

Incremental Analysis for Multiple Alternatives (cont.)

4) Compare the winner (Defender) to the next more costly alternative (Challenger), each round leaving one winner standing.
5) Keep doing step 4 until we exhaust all alternatives. Select the final winner.

Note that for multiple independent alternatives, an incremental analysis is NOT needed.

- Without a budget limitation, select all projects whose ROR > MARR.
- An incremental analysis is required for selecting among mutually-exclusive projects using ROR.

The Initial Example Revisited

Ex Imagine two investments:

A: Invest $100, pays $180 one year from now

B: Invest $1,000,000, pays $1,500,000 one year from now

Which one would you choose?

The Initial Example Revisited (cont.)

Note that using an incremental analysis will give the correct answer. (Just comparing ROR_A with ROR_B is not correct.)

End of Slide Set

Topic 8
Breakeven Analysis: Single and Multiple Alternatives

Roadmap

Imagine a business idea. How do you know if it is profitable?

1) When is Breakeven Analysis Useful?

- Single alternative: when is the project profitable or acceptable?
- Two or more alternatives: when is one alternative better than the other(s)?
- a popular application of BE analysis is the cost-revenue-volume relationships. (find BE quantity)

2) Cost Models: Fixed Costs

- Fixed costs do not vary with the production or activity levels.
- Examples:
  - cost of building
  - insurance
  - fixed overhead
  - equipment capital recovery
- Fixed costs still incur even at no level of activity.
- The facility or operation must be shutdown in order to lower or eliminate fixed costs.

Cost Models: Variable Costs

- Variable costs vary with the level of activity.
- Examples:
  - direct labor (wages)
  - Material cost
  - indirect costs
  - marketing / advertising
  - warranty
- The more activity, the greater variable costs.
- Variable costs can be lowered with higher operational efficiency, improved design, and higher sales volume.

Cost Models: Total Cost and Profit

Total Cost = Fixed Costs + Variable Costs

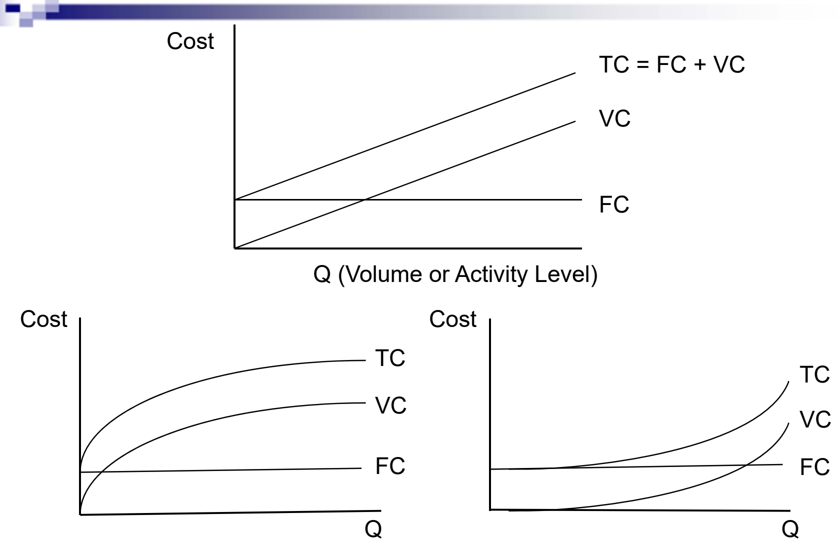
$$TC = FC + VC$$

Profit = Revenue - Total Cost

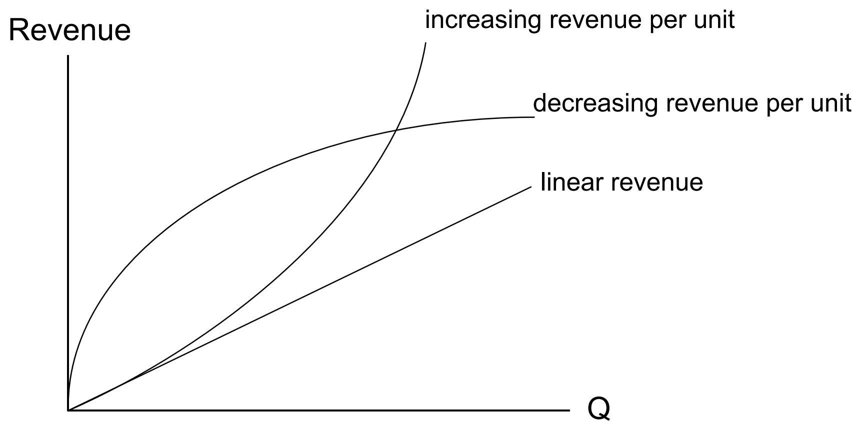
$$P = R - TC$$

$$P = R - (FC + VC)$$

Cost Functions



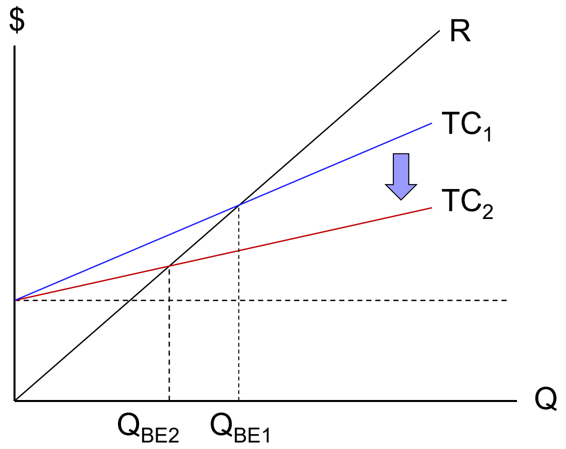
Revenue Functions



3) Breakeven Analysis for a Single Alternative

- A breakeven value is determined by setting PV, FV or AV = 0 and solve for the unknown parameter.
- For a revenue/cost problem, Q_{BE} is the point where revenue = cost (revenue intersects cost).
- Over this point, the project becomes profitable. Under this point, the project results in a loss.
- Breakeven points can be useful for planning purposes.

Breakeven Analysis for a Single Alternative (cont.)



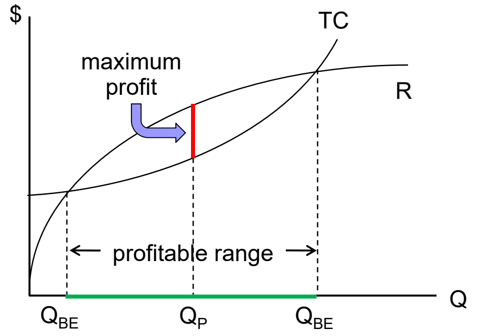
If a variable cost is reduced, Q_{BE} will be lower.

Why?

Example: BE Analysis for a Single Alternative

Ex You intend to start a new business with an initial setup cost of $400,000 and a salvage value of $50,000 after five years. If a variable cost of production is $30/unit and a selling price is $50/unit. At MARR of 1% per month, how many units do you have to sell each month to break even?

Non-linear Cost and Revenue Models



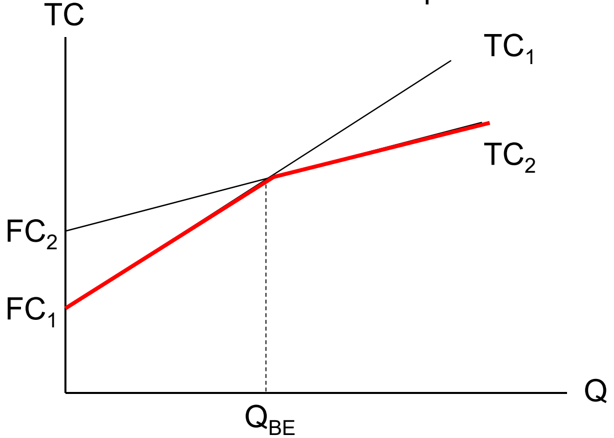
For non-linear functions, there may exist multiple breakeven points.

4) Breakeven Analysis for Two Alternatives

- Given two mutually-exclusive alternatives, find the breakeven point over which one alternative is better than the other or where you are indifferent between the two alternatives.
- Procedure:
  1) Determine a common variable, such as an interest rate, an investment cost, AOC, production quantity, etc.
  2) Formulate PV or AV relationships of each alternative (or any economic measure with the common variable in the expressions).
  3) Set the two economic expressions equal and solve for the common variable (BE point).

Breakeven Analysis for Two Alternatives (cont.)

Total cost relationships of two alternatives and the breakeven point



Alt. 1: cheaper FC, higher VC
Alt. 2: more expensive FC, lower VC

Example: Breakeven Analysis for Two Alternatives

<u>Ex</u> A factory is considering purchasing a new food processing machine. A manual machine costs $50,000 upfront and will last 10 years with no salvage value. An automatic machine costs $100,000 and, due to several electronic components, will only last 8 years with no salvage value. Compared to the automatic machine, the manual machine requires an extra $50 of labor to preprocess each ton of food. At MARR = 12%, how many tons of food must the factory process each year in order to justify buying the automatic machine?

Example: Breakeven Analysis for Two Alternatives (cont.)

Example: Breakeven Analysis Using Incremental Analysis

Ex Two alternatives for water heater systems are available:

1) Solar heating system requires an initial investment in a solar collector, a storage tank and copper pipes of $60,000. With the system, the facility's electricity cost will be $84,000 per year.
2) Standard electricity-powered heaters require an initial cost of $32,000, but the facility's electricity cost will be $90,000 per year.

At MARR of 6%, how long must the facility be needed in order to justify an investment in the solar heating system?

Example: Breakeven Analysis Using Incremental Analysis (cont.)

- The objective is to find a breakeven life between two alternatives.
- Approach: formulate incremental cash flows to find the difference between two alternatives.
- Determine PV(6%), where "n" is treated as a varying parameter.
- Breakeven life is where PV of incremental cash flows changes from negative to positive.

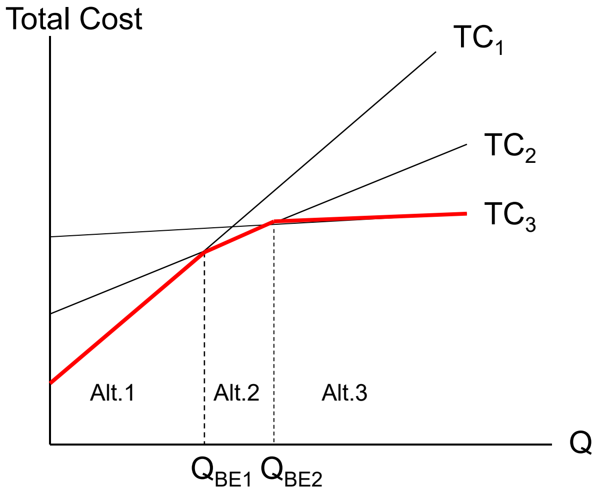
Example: Breakeven Analysis Using Incremental Analysis (cont.)

Interest rate = 6%

t	Standard Electricity-powered System (A)	Solar Heating System (B)	(B-A)	PV for "n" years (from t=0 to t=n)
0	$ (32,000)	$ (60,000)	$ (28,000)	$ (28,000.00)
1	$ (90,000)	$ (84,000)	$ 6,000	$ (22,339.62)
2	$ (90,000)	$ (84,000)	$ 6,000	$ (16,999.64)
3	$ (90,000)	$ (84,000)	$ 6,000	$ (11,961.93)
4	$ (90,000)	$ (84,000)	$ 6,000	$ (7,209.37)
5	$ (90,000)	$ (84,000)	$ 6,000	$ (2,725.82)
6	$ (90,000)	$ (84,000)	$ 6,000	$ 1,503.95
7	$ (90,000)	$ (84,000)	$ 6,000	$ 5,494.29
8	$ (90,000)	$ (84,000)	$ 6,000	$ 9,258.76

PV changes from (-) to (+) between n = 5 and n = 6

5) Breakeven Analysis for Multiple Alternatives



1) Compare alternatives pairwise.
2) Find BE points.
3) Determine a lower envelope (cost model) or an upper envelope (profit model).

Example: Breakeven Analysis for Multiple Alternatives

<u>Ex</u> A company is considering among three alternatives for its part procurement:

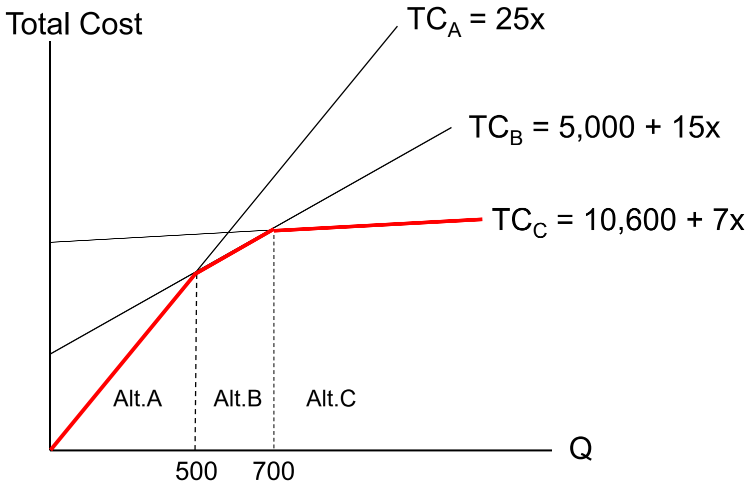
A. Buy from a supplier at $25/piece
B. Co-invest with an outsource contractor, which requires a monthly fixed cost of $5,000 but will allow the parts to be obtained at $15/piece
C. Manufacture the parts in house at a monthly fixed cost of $10,600 and a variable cost of $7/piece

Which alternative is the best and at what demand level of parts (pieces/month)?

Example: Breakeven Analysis for Multiple Alternatives (cont.)

Example: Breakeven Analysis for Multiple Alternatives (cont.)

Example: Breakeven Analysis for Multiple Alternatives (cont.)



6) Spreadsheet Models

- When the models are non-linear or the cash flows are complicated, use spreadsheet to help.
- Excel has a built-in analysis tool, such as solver.
- Solver can help with “goal seek” or “what-if” analysis.
- “Goal seek” = set “target cell” (max, min, equal to) by “changing cells” (decision variables).
- Solver iterates over various decision values in order to the find the desired target value.

6) Spreadsheet Models

For breakeven analysis:

- Single alternative decision: Set economic measure: PV, AV = 0 or ROR = MARR (target cell) - by changing a decision variable.
- Two-alternative decision: set the economic measure (PV, AV, etc.) of the two alternatives equal, find the decision variable.
- Solver may
  - find the exact solution
  - find the closest solution
  - cannot find a feasible solution

End of Slide Set

Topic 9
Decision under Uncertainties:
Analysis Techniques

Roadmap

1) Sensitivity Analysis
2) Three Estimates
3) Expected Value
4) Decision Tree

Dealing with Uncertainties

- In earlier chapters, all parameters are assumed to be known with certainty, such as cash flows, MARR, life, etc.
- What if these are uncertain?
- Think of an investment. What could be uncertain? How do you deal with these uncertainties?
- Besides breakeven analysis, there are various techniques to address uncertainties.

1) Sensitivity Analysis

- In engineering economics, parameters that may be uncertain include: initial investment, future costs, future revenue/income, salvage value, MARR, life, etc. (P, F, A, i, n)
- Sensitivity analysis determines how much an output changes when one or more input parameters change.
- If a small change in an input parameter leads to a large change in an output variable, the output is *sensitive* to an input. Otherwise, an output is *insensitive* to an input.

Importance of Sensitivity

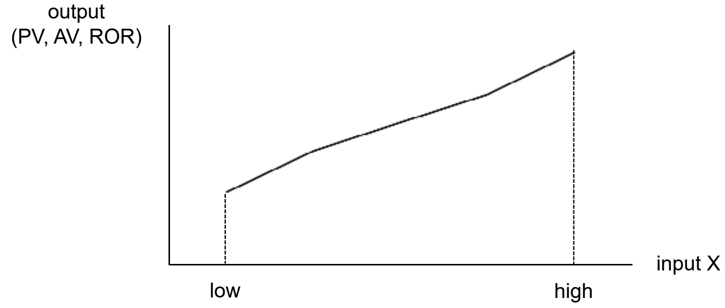
- In engineering economic applications, sensitivity analysis should be performed on key inputs.
- A parameter, to which an outcome is <u>sensitive</u>, deserves an additional effort to estimate more precisely.
- Sensitivity analysis also shows the degree of risks associated with a project (how an outcome varies over a feasible range of inputs).

Sensitivity in Engineering Economics

Sensitivity analysis determines

1) how an economic measure (PV, AV, ROR, etc.)
2) the best alternative

change as the input value changes.



Sensitivity Analysis Procedure

1) Determine input parameters that may vary.
2) Define a possible range of values and an incremental step.
3) Select an economic measure (output).
4) For each input parameter, compute an output value over a possible range of input values.
5) Graph the results for interpretation; x-axis is the input and y-axis is the output.

<u>Interpretation</u>: The steeper the slope, the more sensitive the output is to the input.

Example: Sensitivity Analysis

<u>Ex</u> An equipment investment to enhance the production requires \$100,000 upfront and will generate an additional income of \$15,000 – 35,000 per year. It has a useful life of 6 – 10 years. At MARR of 12%, explore sensitivity of the project’s PV to annual income and the equipment life.

Sensitivity of PV to Annual Income

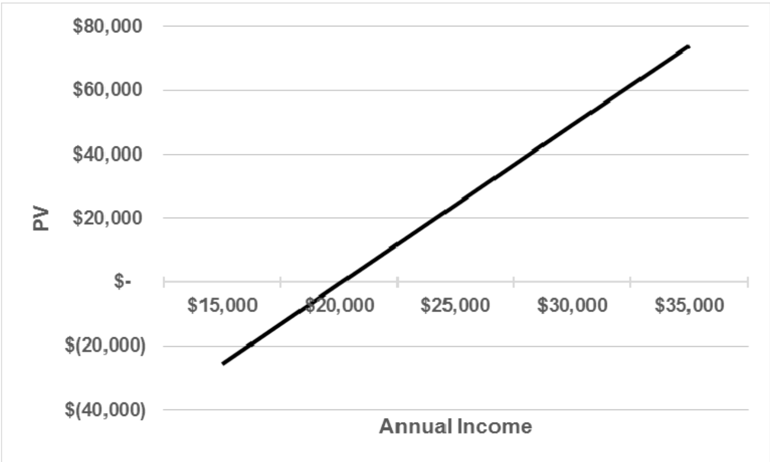
- Input parameter = annual income
- Range = $15,000 – $35,000
- Incremental step = $5,000
- An economic measure = PV
- Fix the equipment life to 8 years

Sensitivity of PV to Annual Income

- MARR = 12%
- Investment cost = $100,000
- Annual income = $15,000 - $35,000

Annual Income	PV
$15,000	- $25,485
$20,000	- $647
$25,000	$24,191
$30,000	$49,029
$35,000	$73,867

Sensitivity of PV to Annual Income



Sensitivity of PV to Life

- Input parameter = equipment life
- Range = 6 – 10 years
- Incremental step = 1 year
- An economic measure = PV
- Fix the annual income to $25,000/year

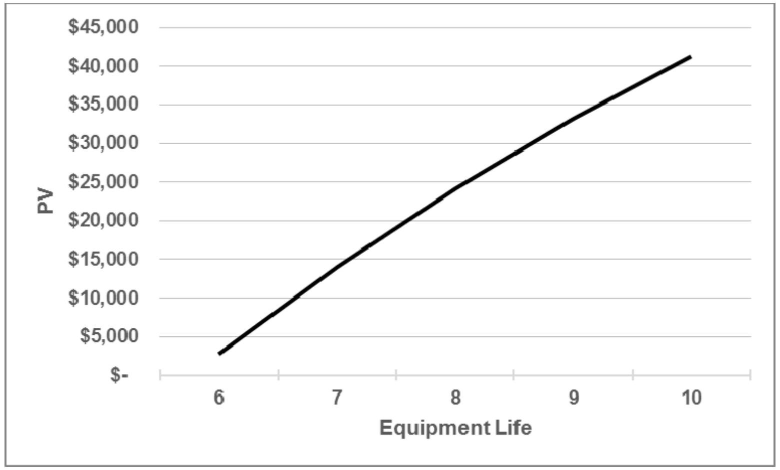
Sensitivity of PV to Life

Year	Cash Flow
0	- 100,000
1	25,000
2	25,000
3	25,000
4	25,000
5	25,000
6	25,000
7	25,000
8	25,000
9	25,000
10	25,000

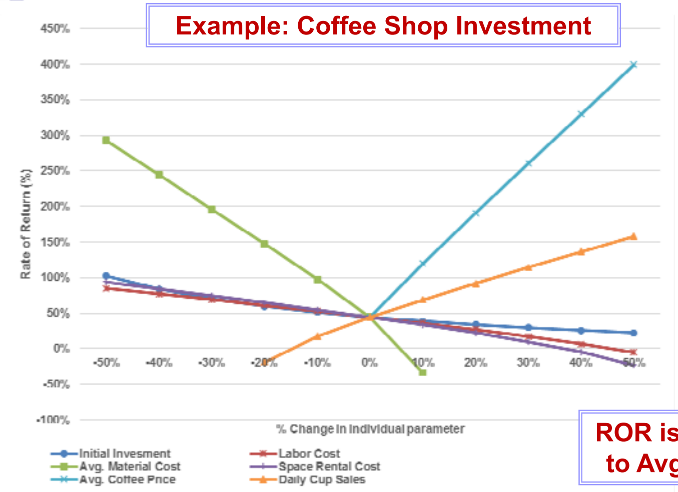
- MARR = 12%
- Investment cost = $100,000
- Annual income = $25,000

Equipment Life	PV
6	$2,785
7	$14,094
8	$24,191
9	$33,206
10	$41,256

Sensitivity of PV to Life



Sensitivity to Multiple Parameters



The greater the slope, the more sensitive the output (ROR) is to that input variable.

ROR is most sensitive to Avg. Coffee Price.

2) Three Estimates (Scenario Analysis)

- Making three estimates for each parameter
  - Pessimistic value (P)
  - Most likely value (ML)
  - Optimistic value (O)
- For each scenario, several parameters change their values simultaneously.
- Depending on the nature of parameters, pessimistic (or optimistic) values could be the lowest or highest values.

Example: Three Estimates

Ex Two candidate technologies are available with the following estimates. At MARR = 12%, which one should be selected?

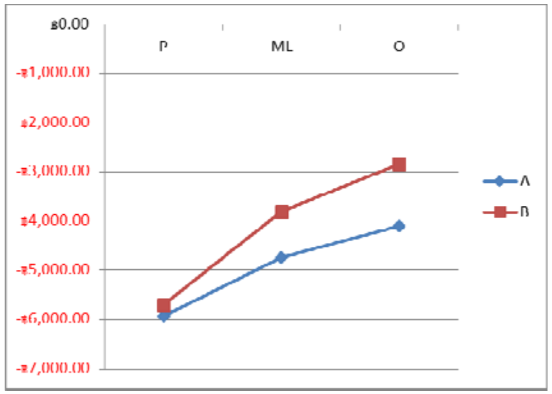
Technology	Scenario	First Cost	SV	AOC	Life
A					
	P	-10000	1000	-4000	8
	ML	-10000	2000	-3000	9
	O	-10000	3000	-2500	10
B					
	P	-15000	3000	-2000	5
	ML	-15000	5000	-1000	7
	O	-15000	7000	-500	9

Example: Three Estimates (cont.)

- For each scenario, compare using AV of cost at 12%
- AV = FC(A/P, 12%, n) + AOC + SV(A/F, 12%, n)

Technology	Scenario	First Cost	SV	AOC	Life	AW
A						
	P	-10000	1000	-4000	8	-฿5,931.73
	ML	-10000	2000	-3000	9	-฿4,741.43
	O	-10000	3000	-2500	10	-฿4,098.89
B						
	P	-15000	3000	-2000	5	-฿5,688.92
	ML	-15000	5000	-1000	7	-฿3,791.18
	O	-15000	7000	-500	9	-฿2,841.43

Example: Three Estimates (cont.)



Which alternative is better?

Why?

3) Expected Value

- Model uncertainty using probability distribution
- Probability assignment requires experience and experts judgment.
- Use an expected value or a mean as a representative of the outcomes.

Definition of Expected Value

$$E(X) = \sum_{i=1}^{n} X_i P(X_i)$$

- X_i = a possible outcome of a variable X
- n = the number of possible outcomes X can assume
- $P(X_i)$ = the probability that X_i occurs

$$\sum_{i=1}^{n} P(X_i) = 1$$

(Exhaust all possible outcomes)

Example: Expected Value

Ex A company is deciding whether to invest $1M in a particular oil exploration. The possible three outcomes of revenue along with their probabilities, depending on the oil flow rate, are:

t	High	Medium	Low
0	$(1,000,000)	$(1,000,000)	$(1,000,000)
1	$800,000	$ 600,000	$400,000
2	$600,000	$500,000	$300,000
3	$500,000	$300,000	$200,000
PV(15%)	$478,096	$97,066	$(293,828)
Probability	0.3	0.5	0.2

Example: Expected Value (cont.)

- At MARR = 15%, the expected value of PV of this investment:

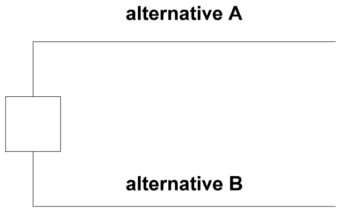
EV = 0.3(478,096) + 0.5(97,066) + 0.2(-293,828)
= $133,196

- Worth investing, as PV(15%) is positive.
- Note that $133,196 is not one of the possible outcomes, but it represents the weighted average (mean) of all outcomes (when the weights are their corresponding probabilities).

4) Decision Tree

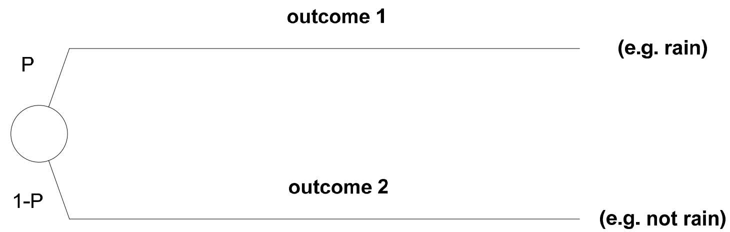
- Helps frame "staged decisions"
- Results from this stage leads to a decision or uncertainty in the next stage.
- The tree is constructed from left to right by branching out nodes. There two types of nodes: decision node and probability node.
- Evaluation is done from right to left, given the value of all end results.

Decision Node



- Use <u>a square</u> to represent a decision node.
- Branches are possible alternatives.
- The <u>value</u> of a decision node is the value of the best alternatives (value = max[alt. A, alt. B, …])

Probability Node

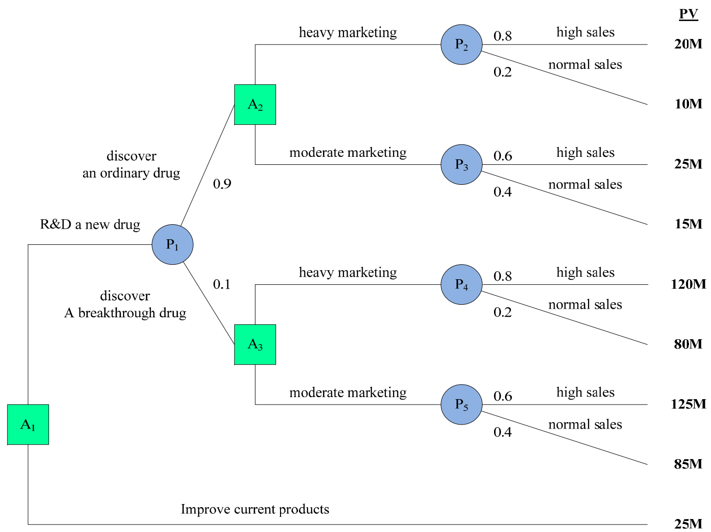


- Use <u>a circle</u> to represent a probability node.
- Branches are possible outcomes of an uncertainty.
- Probability of each outcome is written on a branch.
- The <u>value</u> of a probability node is <u>an expected value</u> of the outcomes (value = E[outcomes])

Example: Decision Tree

<u>Ex</u> A pharmaceutical company is deciding whether to spend its remaining budget on R&D of a new drug or on an improvement of its current products. The project to improve current products will yield a present value of 25M with certainty. Over the same time period, R&D of a new drug entails many uncertainties. There is 10% chance of discovering a breakthrough drug and 90% chance of resulting in an ordinary drug. After a new drug discovery, the company also has to decide whether to undertake heavy marketing campaign of the new drug which costs 5M more but also leads to a better chance of high sales. The corresponding PV, inclusive of all costs and revenue, of each outcome is shown. What should the company do?

Example: Decision Tree (cont.)



Example: Decision Tree (cont.)

- From the evaluation, the company should R&D a new drug.
- Also, if R&D results in an ordinary drug, it should pursue a moderate marketing campaign. If R&D results in a breakthrough drug, it is optimal to conduct a heavy marketing campaign.
- The value of this decision is 30.1M

Final Note on Decision Tree

- Useful framework in thinking about a decision
- Learn a lot just by constructing the tree and understanding the consequences and complications of a decision/uncertainty.
- The analysis will provide the best decision policy at each stage, given the previous outcomes.

End of Slide Set